Solutions to New MFE Sample Exam Questions

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In February 2009, the SOA/CAS released 18 additional sample exam questions for MFE/3F, making a total of 49 questions.[1] In addition the solution to question #23 was rewritten slightly.

32. Would go in my Section 45 on the Geometric Brownian Motion Model of Stock Prices.
33. Would go in my Section 40 on Other Exotic Options.
34. Would go in my Section 42 on Standard Brownian Motion.
35. Would go in my Section 47 on Ito's Lemma.
36. Would go in my Section 48 on Valuing a Claim on S^a.
37. Would go in my Section 44 on Geometric Brownian Motion. See Q. 43.28 and Q. 44.10.
38. Would go in my Section 49. See also my Sections 51 and 52.
39. Would go in my Section 49. See also my Sections 4, 55, and 56.
40. Would go in my Section 28 on Profit on Options Prior to Expiration. See also my Section 1.
41. Would go in my Section 27 on Elasticity and Related Ideas.
42. Would go in my Section 36 on Barrier Options.
43. Would go in my Section 47 on Ito's Lemma.
44. Would go in my Section 13 on Binomial Trees.
45. Would go in my Supplement, under Greeks in the Binomial Model (Appendix 13.B).
46. Would go in my Section 14 on Binomial Trees, Valuing Options on Other Assets.
47. Would go in my Section 28 on Profit on Options Prior to Expiration.
48. Would go in my Section 46 on Ito Processes.
49. Would go in my Section 13 on Binomial Trees.

[1] Check the SOA webpage for any updates.

32. At time $t = 0$, Jane invests the amount of $W(0)$ in a mutual fund. The mutual fund employs a proportional investment strategy: There is a fixed real number φ, such that, at every point of time, $100\varphi\%$ of the fund's assets are invested in a nondividend paying stock and $100(1 - \varphi)\%$ in a risk-free asset. You are given:

(i) The continuously compounded rate of return on the risk-free asset is r.

(ii) The price of the stock, $S(t)$, follows a geometric Brownian motion,

$$dS(t)/S(t) = \alpha\, dt + \sigma\, dZ(t),\quad t \geq 0.$$

where $\{Z(t)\}$ is a standard Brownian motion.

Let $W(t)$ denote the Jane's fund value at time t, $t \geq 0$.

Which of the following equations is true?

(A) $dW(t)/ W(t) = \{\varphi\alpha + (1 - \varphi)r\}dt + \sigma\, dZ(t)$

(B) $W(t) = W(0) \exp\{[\varphi\alpha + (1 - \varphi)r]t + \sigma Z(t)\}$

(C) $W(t) = W(0) \exp\{[\varphi\alpha + (1 - \varphi)r - \varphi\sigma^2/2]t + \sigma Z(t)\}$

(D) $W(t) = W(0) \{S(t)/S(0)\}^{\varphi} e^{(1-\varphi)rt}$

(E) $W(t) = W(0) \{S(t)/S(0)\}^{\varphi} \exp[(1 - \varphi)(r + \varphi\sigma^2/2)t]$

32. E. At time t, the fund owns x shares of stock and bonds worth y.
Then $W(t) = x(t)\,S(t) + y(t)$. $\varphi = x(t)\,S(t)/W(t)$. $x(t) = \varphi W(t)/S(t)$.
$y(t) = W(t) - x(t)\,S(t) = (1 - \varphi)W(t)$.
Then, since the stock pays no dividends, $dW = x dS + y r dt = \varphi W(t)/S(t)\, dS + (1 - \varphi)W(t) r\, dt$.
$\Rightarrow dW/W = \varphi\, dS/S + (1 - \varphi) r\, dt$.
In other words, the instantaneous return on the fund is a weighted average of the return on the stock and the risk free rate.
$dS/S = \alpha\, dt + \sigma\, dZ$. $\Rightarrow$
$dW/W = \varphi \{\alpha\, dt + \sigma\, dZ\} + (1 - \varphi) r\, dt = \{\varphi\, \alpha + (1 - \varphi) r\}\, dt + \varphi\sigma\, dZ$.
Therefore, W(t) is a Geometric Brownian Motion, with $\sigma' = \varphi\sigma$, and $\mu + \sigma'^2/2 = \varphi\, \alpha + (1 - \varphi) r$
Therefore, $W(t)/W(0) = \exp[\{\varphi\, \alpha + (1 - \varphi) r - \varphi^2\sigma^2/2\}t + \varphi\sigma Z]$.
Since S(t) is a Geometric Brownian Motion: $S(t)/S(0) = \exp[(\alpha - \sigma^2/2)t + \sigma Z]$.
$W(t)/W(0) = \exp[\{(1 - \varphi) r + \varphi\sigma^2/2 - \varphi^2\sigma^2/2\}t + \varphi\{(\alpha - \sigma^2/2)t + \sigma Z\}] =$
$\mathbf{\{S(t)/S(0)\}^{\varphi} \exp[(1 - \varphi)(r + \varphi\sigma^2/2)t]}$.
<u>Comment</u>: The mutual fund's portfolio has to be continuously rebalanced.
For example, if the stock price increases by a lot, then more than φ of the funds assets will be in stock. Thus the fund would need to sell some stock and buy some risk free bonds.
$W(t)/W(0) = \{S(t)/S(0)\}^{\varphi} \exp[rt]^{1-\varphi} \exp[(1 - \varphi)\varphi(\sigma^2/2)t]$.
$\ln[W(t)/W(0)] = \varphi \ln[S(t)/S(0)] + (1 - \varphi)(rt) + (1 - \varphi)\varphi(\sigma^2/2)t$
$= \varphi$(return on the stock) + $(1 - \varphi)$(return on risk free asset) + $(1 - \varphi)\varphi\sigma^2/2)t$.

33. You own one share of a nondividend-paying stock. Because you worry that its price may drop over the next year, you decide to employ a *rolling insurance strategy*, which entails obtaining one 3-month European put option on the stock every three months, with the first one being bought immediately.

You are given:

(i) The continuously compounded risk-free interest rate is 8%.

(ii) The stock's volatility is 30%.

(iii) The current stock price is 45.

(v) The strike price for each option is 90% of the then-current stock price.

Your broker will sell you the four options but will charge you for their total cost now.

Under the Black-Scholes framework, how much do you now pay your broker?

(A) 1.59 (B) 2.24 (C) 2.85 (D) 3.48 (E) 3.61

33. C. For a forward start put, with forward start date t_1, expiration time T, with strike price 90% of the current stock price, the premium is:

$S_0 \exp[-\delta\, t_1]$ {90% $\exp[-r(T - t_1)]\, \Phi[-d_2] - \exp[-\delta(T - t_1)]\, \Phi[-d_1]$},

where $d_1 = \{-\ln[90\%] + (r - \delta + \sigma^2/2)(T - t_1)\}/\{\sigma \sqrt{(T - t_1)}\}$,

and $d_2 = d_1 - \sigma \sqrt{(T - t_1)}$.

$d_1 = \{-\ln[90\%] + (8\% - 0 + .3^2/2)(1/4)\}/\{.3 \sqrt{(1/4)}\} = 0.9107$. $d_2 = 0.9107 - .3 \sqrt{(1/4)} = 0.7607$.

Premium for the first forward start put is:

45 {90% $\exp[-8\%/4]\, \Phi[-0.76] - \exp[0]\, \Phi[-.91]$} = (45){(0.8822)(.2236) - 0.1814} = 0.7137.

The premium for the second forward start put would be that for the first put times $\exp[-\delta/4]$.

Since $\delta = 0$, the premiums are the same, as are those of the other two puts.

Therefore, the premium for the rolling insurance strategy is: (4)(0.7137) = **2.85**.

Comment: See Problem 14.22 in Derivative Markets by McDonald.

34. The cubic variation of the standard Brownian motion for the time interval [0, T] is defined analogously to the quadratic variation as

$$\lim_{n \to \infty} \sum_{j=1}^{j=n} \{Z[j\,h] - Z[(j-1)\,h]\}^3,$$

where h = T/n.

What is the distribution of the cubic variation?

(A) N(0, 0)

(B) $N(0, T^{1/2})$

(C) N(0, T)

(D) $N(0, T^{3/2})$

(E) $N(-\sqrt{(T/2)}, T)$

34. A. For a random walk, each cubed increment is equal to the cube of the size of the step. If on (0, T) we have n steps each of size $\pm\sqrt{h} = \pm\sqrt{(T/n)}$, then

$$\lim_{n \to \infty} \sum_{j=1}^{n} |\, Z(jT/n) - Z((j-1)T/n) \,|^3 = \lim_{n \to \infty} (n)(T/n)^{3/2} = \lim_{n \to \infty} T^{3/2}/\sqrt{n} = 0.$$

$$\left|\sum_{j=1}^{j=n} \{Z[j\,h] - Z[(j-1)\,h]\}^3\right| \le \sum_{j=1}^{n} |\, Z(jT/n) - Z((j-1)T/n) \,|^3.$$

Therefore, as $n \to \infty$, the cubic variation of a random walk goes to zero.

Since a Brownian motion is a limit of random walks, it also has a cubic variation of zero.

This is the same as a Normal Distribution with mean 0 and standard deviation 0, ***N*****(0, 0)**, in other words always zero.

Comment: At page 653, McDonald states that “higher-order [than quadratic] variations are zero. Thus for example, the sum of the cubed increments is zero.”

35. The stochastic process $\{R(t)\}$ is given by

$$R(t) = R(0)e^{-t} + 0.05(1 - e^{-t}) + 0.1 \int_0^t e^{s-t} \sqrt{R(s)}\, dZ(s),$$

where $\{Z(t)\}$ is a standard Brownian motion.

Define $X(t) = [R(t)]^2$.

Find $dX(t)$.

(A) $\{0.1 \sqrt{X(t)} - 2X(t)\}dt + 0.2\, X(t)^{3/4}\, dZ(t)$

(B) $\{0.11 \sqrt{X(t)} - 2X(t)\}dt + 0.2\, X(t)^{3/4}\, dZ(t)$

(C) $\{0.12 \sqrt{X(t)} - 2X(t)\}dt + 0.2\, X(t)^{3/4}\, dZ(t)$

(D) $\{0.01 + (0.1 - 2\, R(0))e^{-t}\}\sqrt{X(t)}\, dt + 0.2\, X(t)^{3/4}\, dZ(t)$

(E) $\{0.1 - 2\, R(0)\}e^{-t} \sqrt{X(t)}\, dt + 0.2\, X(t)^{3/4}\, dZ(t)$

35. B. $R(t) = R(0)e^{-t} + 0.05(1 - e^{-t}) + 0.1e^{-t}\int_0^t e^s \sqrt{R(s)}\, dZ(s)$.

$dR = -R(0)e^{-t}dt + 0.05e^{-t}dt - 0.1e^{-t}\, dt\int_0^t e^s \sqrt{R(s)}\, dZ(s) + 0.1e^{-t}\, e^t\sqrt{R}\, dZ$.

$dR = (0.05 - R)dt + 0.1\sqrt{R}\, dZ$.

$dR^2 = (0.05 - R)^2dt^2 + 2(0.05 - R)dt0.1\sqrt{R}\, dZ + 0.1^2R\, dZ^2 = 0.01\, R\, dt$.

$X = R^2$. $\partial X/\partial R = 2R$. $\partial^2X/\partial R^2 = 2$. $\partial X/\partial t = 0$.

By Ito's Lemma, $dX = \partial X/\partial R\, dR + \partial^2X/\partial R^2\, dR^2/2 + \partial X/\partial t\, dt =$

$2R\{(0.05 - R)dt + 0.1\sqrt{R}\, dZ\} + (2)\, 0.01\, R\, dt/2 =$

$0.1Rdt - 2R^2dt + 0.2R^{3/2}\, dZ + 0.01\, R\, dt =$

$\mathbf{\{0.11\sqrt{X} - 2X\}dt + 0.2\, X^{3/4}\, dZ}$.

Comment: See Problem 20.9 in Derivative Markets by McDonald.

From calculus, $d\int_{f(t)}^{g(t)} h(s)\, ds/\, dt = g'(t)\, h(g(t)) - f'(t)\, h(f(t))$.

$dR = (0.05 - R)dt + 0.1\sqrt{R}\, dZ$, is the form of a Cox-Ingersoll-Ross interest rate model.

36. Assume the Black-Scholes framework. Consider a derivative security of a stock.
You are given:
(i) The continuously compounded risk-free interest rate is 0.04.
(ii) The volatility of the stock is σ.
(iii) The stock does not pay dividends.
(iv) The derivative security also does not pay dividends.
(v) S(t) denotes the time-t price of the stock.
(iv) The time-t price of the derivative security is $[S(t)]^{-k/\sigma^2}$, where k is a positive constant.

Find k.

(A) 0.04 (B) 0.05 (C) 0.06 (D) 0.07 (E) 0.08

36. E. $V = S^{-k/\sigma^2}$. $V_t = 0$. $V_S = \partial V/\partial S = (-k/\sigma^2)\, S^{-k/\sigma^2 - 1} = (-k/\sigma^2)V/S$.

$V_{SS} = \partial^2 V/\partial S^2 = (-k/\sigma^2)(-k/\sigma^2 - 1)S^{-k/\sigma^2 - 2} = (k/\sigma^2)(k/\sigma^2 + 1)V/S^2$.

There are no dividends, and therefore by the Black-Scholes Equation:

$V_t + \sigma^2 S^2 V_{SS}/2 + r\,S\,V_S - rV = 0. \Rightarrow$

$0 + k(k/\sigma^2 + 1)V/2 - r(k/\sigma^2)V - rV = 0. \Rightarrow$

$k^2 + (\sigma^2 - 2r)k - 2r\sigma^2 = 0. \Rightarrow$

$k = \{2r - \sigma^2 \pm \sqrt{\{(\sigma^2 - 2r)^2 - 8r\sigma^2\}}\}/2 = \{2r - \sigma^2 \pm (\sigma^2 + 2r)\}/2 = 2r$ or $-\sigma^2$.

However, we are told k is positive, so that k = 2r = (2)(.04) = **0.08**.

Alternately, since the derivative pays no dividends, its prepaid forward price is its current price.

Let $a = -k/\sigma^2$. Then $S_0{}^a$ is the current price of this derivative.

We know that for a claim that pays $S_T{}^a$, the prepaid forward price is:

$e^{-rT}\, S_0{}^a \exp[\{a(r - \delta) + a(a-1)\sigma^2/2\}T]$.

Therefore, $S_0{}^a = e^{-.04T}\, S_0{}^a \exp[\{a(.04) + a(a-1)\sigma^2/2\}T]$.

$\Rightarrow 1 = \exp[\{-.04 + a(.04) + a(a-1)\sigma^2/2\}T]$

$\Rightarrow 0 = -.04 + a(.04) + a(a-1)\sigma^2/2. \Rightarrow a^2\sigma^2 + a(.08 - \sigma^2) - .08 = 0.$

$\Rightarrow a = \{\sigma^2 - .08 \pm \sqrt{\{(\sigma^2 - .08)^2 + .32\sigma^2\}}\}/(2\sigma^2) = \{\sigma^2 - .08 \pm (\sigma^2 + .08)\}/(2\sigma^2) = -0.08/\sigma^2$ or 1.

$\Rightarrow k = 0.08$ or $-\sigma^2$. However, we are told k is positive, so that k = **0.08**.

Comment: If the derivative paid dividends, then it would not satisfy Black-Scholes Equation.
As in Problem 21.10 in Derivative Markets by McDonald, the equation would have to be modified.
If the derivative makes continuous payments at a rate γ, in others words over time period dt the derivative pays γdt, then the equation becomes:

$V_t + \sigma^2 S^2 V_{SS}/2 + (r - \delta)S\,V_S + \gamma - rV = 0.$

37. The price of a stock is governed by the stochastic differential equation:

$dS(t)/S(t) = 0.03dt + 0.2\ dZ(t)$,

where $\{Z(t)\}$ is a standard Brownian motion.

Consider the geometric average: $G = \{S(1)\ S(2)\ S(3)\}^{1/3}$.

Find the variance of ln[G].

(A) 0.03 (B) 0.04 (C) 0.05 (D) 0.06 (E) 0.07

37. D. S follows a Geometric Brownian Motion. $\alpha - \delta = 0.03$, and $\sigma = 0.2$.

$\ln[G] = \{\ln[S(1)] + \ln[S(2)] + \ln[S(3)]\}/3 =$

$\{3\ln[S(1)] + 2(\ln[S(2)] - \ln[S(1)]) + (\ln[S(3)] - \ln[S(2)])\}/3$.

ln[S(1)] is the return from time 0 to 1.

ln[S(2)] - ln[S(1)] is the return from time 1 to 2.

ln[S(3)] - ln[S(2)] is the return from time 2 to 3.

The returns over <u>disjoint</u> time intervals are <u>independent</u>.

Also the variance of the return over a time period of one is: $0.2^2 = 0.04$.

Therefore, $\text{Var}[\ \ln[G]\ ] = \{\text{Var}[3\ln[S(1)]] + \text{Var}[2(\ln[S(2)] - \ln[S(1)])] + \text{Var}[(\ln[S(3)] - \ln[S(2)])]\}/3^2 =$

$\{(9)(0.04) + (4)(0.04) + 0.04\}/9 =$ **0.062**.

<u>Comment</u>: Each of the returns is Normal, and thus so is ln[G].

Therefore, G is LogNormal.

It can be shown that for a geometric average of N stock prices at times: T/N, 2T/N, ... , T,

$\text{Var}[\ln[G]] = (N+1)(2N+1)\sigma^2 T/(6N^2)$.

This is used to derive the formulas for the premiums of Geometric Average Asian options in Appendix 14.A of <u>Derivative Markets</u> by McDonald, <u>not</u> on the syllabus.

38. For $t \leq T$, let P(t, T, r) be the price at time t of a zero-coupon bond that pays 1 at time T, if the short-rate at time t is r.
You are given:
(i) P(t, T, r) = A(t, T) exp[–B(t, T)r] for some functions A(t, T) and B(t, T).
(ii) B(0, 3) = 2.
Based on P(0, 3, 0.05), you use the delta-gamma approximation to estimate P(0, 3, 0.03), and denote the value as $P_{Est}(0, 3, 0.03)$.
Find $P_{Est}(0,3,0.03)$ / P(0, 3, 0.05).

(A) 1.0240 (B) 1.0408 (C) 1.0416 (D) 1.0480 (E) 1.0560

38. B. $P = A\ e^{-Br}$. $\Delta = \partial P/\partial r = -BP$. $\Gamma = \partial\Delta/\partial r = B^2P$.

Δ(0, 3, 0.05) = -2 P(0, 3, 0.05).

Γ(0, 3, 0.05) = (2^2)P(0, 3, 0.05) = 4 P(0, 3, 0.05).

Using the delta-gamma approximation:

P(0, 3, .03) ≅ P(0, 3, 0.05) - 2 P(0, 3, 0.05)(-.02) + 4 P(0, 3, 0.05) $(-.02)^2/2$

= **1.0408** P(0, 3, 0.05).

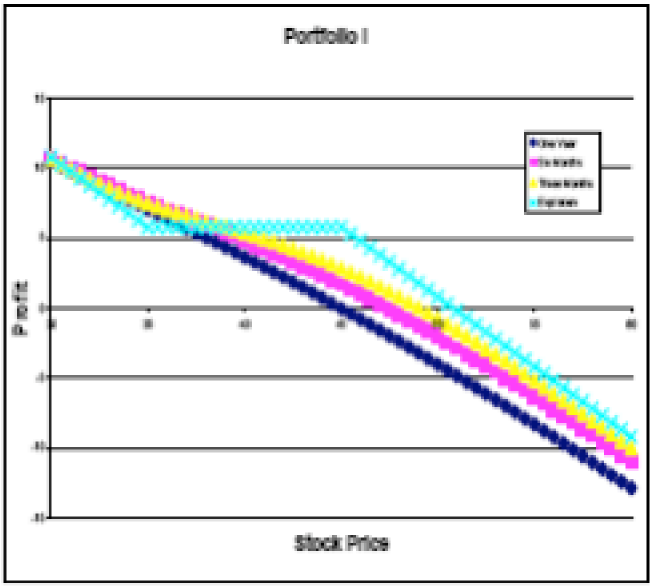
Comment: The delta-gamma approximations for bonds is discussed at page 784 of Derivative Markets by McDonald.
This is the form of the bond price for both the Vasicek and the Cox-Ingersoll-Ross Models.

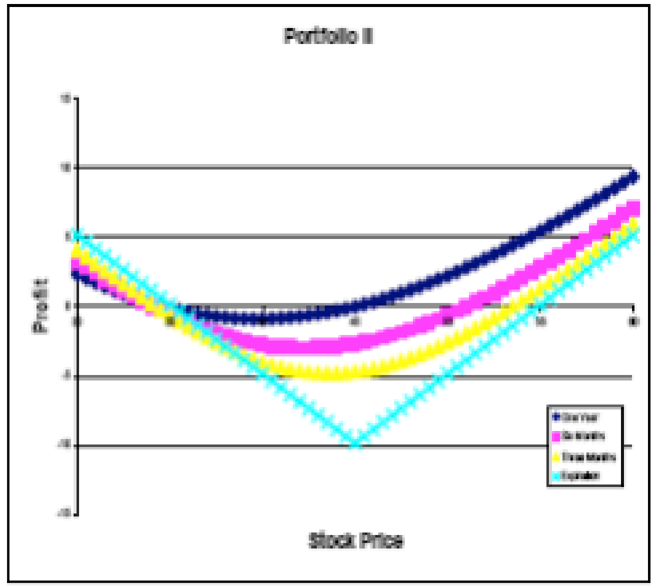
39. A discrete-time model is used to model both the price of a nondividend-paying stock and the short-term interest rate. Each period is one year.
At time 0, the stock price is $S_0 = 100$ and the effective annual interest rate is $r_0 = 5\%$.
At time 1, there are only two states of the world, denoted by u and d.
The stock prices are $S_u = 110$ and $S_d = 95$.
The effective annual interest rates are $r_u = 6\%$ and $r_d = 4\%$.
Let C(K) be the price of a 2-year K-strike European call option on the stock.
Let P(K) be the price of a 2-year K-strike European put option on the stock.
Determine P(108) – C(108).

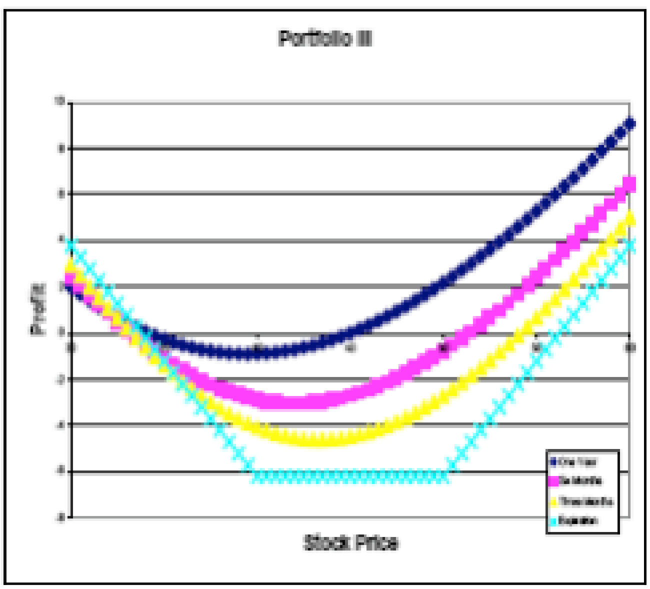
(A) -2.85 (B) -2.34 (C) -2.11 (D) -1.95 (E) -1.08

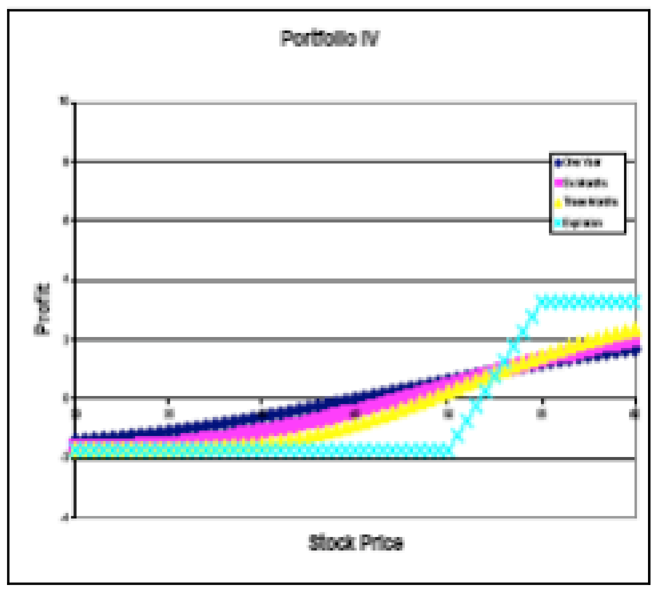
39. B. Assume Clyde bought a two year call and 108 zero coupon bonds that paid 1 at time 2.
Assume Bonnie bought a two year put and a share of stock. (The stock pays no dividends.)
Then if $S_2 > 108$, both Bonnie and Clyde end up with the stock.
Then if $S_2 < 108$, both Bonnie and Clyde end up with 108.
Therefore, Call + 108 P(0, 2) = Put + S_0. A form of put-call parity.
Now p* must be such, that in the risk neutral environment, the stock earns 5%; in this question we use effective annual rates.
$S_0(1.05) = p^*S_u + (1 - p^*)S_d. \Rightarrow$
p* = (105 - 95)/(115 - 95) = 2/3, a version of the usual formula for p*, using effective annual rates.
Therefore, using effective annual rates, the price of a zero coupon bond that pays 1 at time 2 is:
P(0, 2) = p*/{(1.05)(1.06)} + (1 - p*)/{(1.05)(1.04)} = {(2/3)/1.06 + (1/3)/1.04}/1.05 = 0.9042.
Therefore, Call + (108)(0.9042) = Put + 100.
Put Premium - Call Premium = 100 - (108)(0.9042) = **-2.35**.
Even though there are only two states of the world at time 1, we cannot assume that the model is binomial after time 1. However, let us assume any way that we do have a binomial tree, with the same u and d for another year.
Then we get $S_{uu} = 121$, $S_{ud} = 104.5$, and $S_{dd} = 90.25$. However, we need to calculate the different risk neutral probabilities of a move up at the up and down nodes.
At the up node: p* = {(110)(1.06) - 104.5}/(121 - 104.5) = 73.3%.
At the down node: p* = {(95)(1.04) - 90.25}/(104.5 - 90.25) = 60.0%.
Then, the call premium is: (121 - 108)(2/3)(.733)/{(1.05)(1.06)} = 5.71.
The put premium is: (108 - 104.5)(2/3)(.267)/{(1.05)(1.06)} +
(108 - 104.5)(1/3)(.6)/{(1.05)(1.04)} + (108 - 90.25)(1/3)(.4)/{(1.05)(1.04)} = 3.37.
Put Premium - Call Premium = 3.37 - 5.71 = **-2.34**.

40. The following four charts are profit diagrams for four option strategies:
Bull Spread, Collar, Straddle, and Strangle.
Each strategy is constructed with the purchase or sale of two 1-year European options.





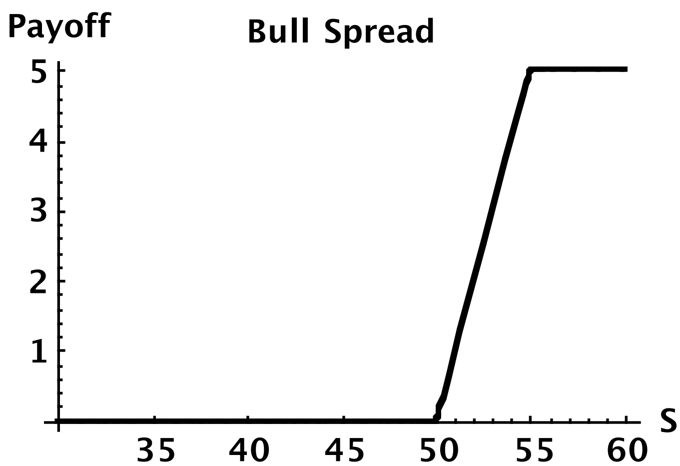




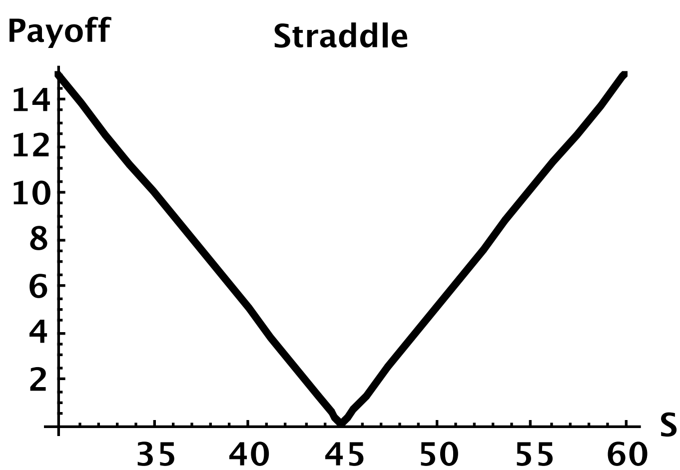
In each chart, the profit is shown with one year, six months, three months, and at expiration.
Match the charts with the option strategies.

	<u>Bull Spread</u>	<u>Straddle</u>	<u>Strangle</u>	<u>Collar</u>
(A)	I	II	III	IV
(B)	I	III	II	IV
(C)	III	IV	I	II
(D)	IV	II	III	I
(E)	IV	III	II	I

40. D. In each case, we need only look at the profit at expiration.
This is the payoff at expiration plus a constant.
(The constant is either the interest we make on a loan or minus the interest we pay on a loan.)
Bull Spread: The purchase of an option together with the sale of an otherwise identical option with a higher strike price.
If the calls have strikes of 50 and 55, then the payoff is: $(S - 50)_+ - (S - 55)_+$.
This is 0 if $S < 50$, $S - 50$ if $50 \le S \le 55$, and 5 if $S > 55$.

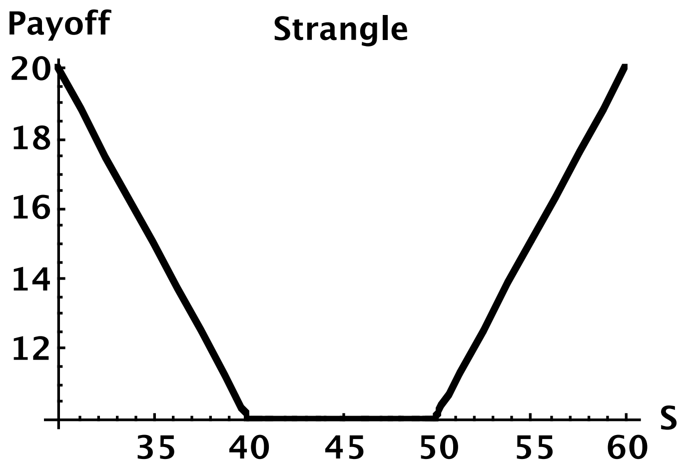


The Bull Spread is graph IV.
Straddle: Buy a put and a call, with the same strike and time until expiration.
If the strike is 45, then the payoff is: $(45 - S)_+ + (S - 45)_+$.
This is $45 - S$ if $S < 45$, and $S - 45$ if $S > 45$.



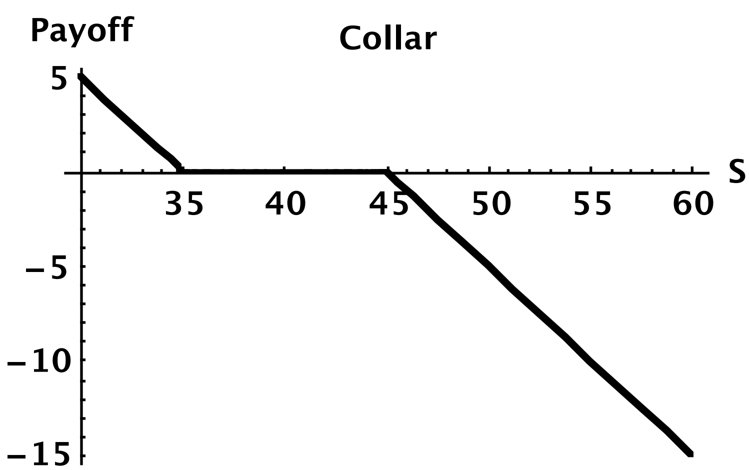
The Straddle is graph II.

Strangle: Buy a put and a higher strike call, with the same time until expiration.
If the put has a strike of 40 and the call has a strike of 50, then the payoff is: $(40 - S)_+ + (S - 50)_+$.
This is 40 - S if S < 40, 0 if 40 ≤ S ≤ 50, and S - 50 if S > 50.



The Strangle is graph III.

Collar: Buy a put and sell a higher strike call, with the same time until expiration.
If the put has a strike of 35 and the call has a strike of 45, then the payoff is: $(35 - S)_+ - (S - 45)_+$.
This is 35 - S if S < 35, 0 if 35 ≤ S ≤ 45, and 45 - S if S > 45.



The Collar is graph I.

<u>Comment</u>: If we purchase a Bull Spread, then we are hoping that the stock price goes up. If instead we purchase a Bear Spread, sale of an option together with buying an otherwise identical option with a higher strike price, then we are hoping that the stock price goes down.

41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock. You are given:

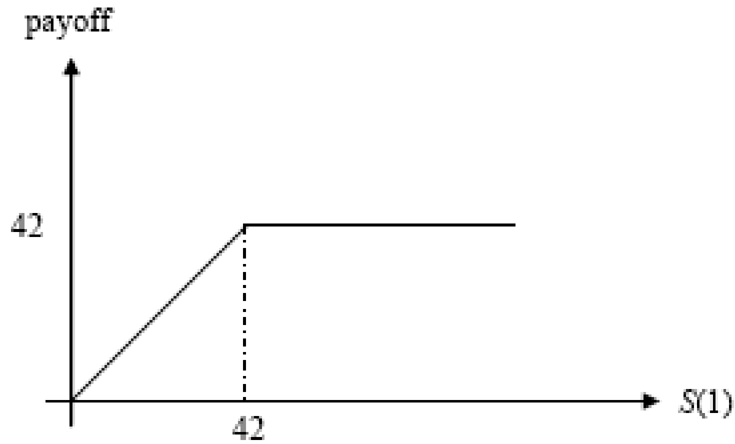
(i) The time-0 stock price is 45.

(ii) The stock's volatility is 25%.

(iii) The stock pays dividends continuously at a rate proportional to its price.
The dividend yield is 3%.

(iv) The continuously compounded risk-free interest rate is 7%.

(v) The time-1 payoff of the contingent claim is as follows:



Calculate the time-0 contingent-claim elasticity.

(A) 0.24 (B) 0.29 (C) 0.34 (D) 0.39 (E) 0.44

41. C. The payoff is S if S < 42, and 42 if S > 42.

The payoff is: $42 - (42 - S)_+ = 42$ - (payoff on a 42 strike put).

$d_1 = \{\ln(S/K) + (r - \delta + \sigma^2/2)T\}/(\sigma\sqrt{T}) =$

$\{\ln(45/42) + (.07 - .03 + .25^2/2)(1)\}/\{(.25)\sqrt{1}\} = 0.5610$

$d_2 = d_1 - \sigma\sqrt{T} = 0.5610 - (.25)\sqrt{1} = 0.3110$. N[.68] = 0.7123. N[.27] = 0.6217.

Put premium is: $K e^{-rT} N[-d_2] - e^{-\delta T} S N[-d_1]$

= (42) exp[-(.07)(1)](1 - 0.6217) - exp[-(0.03)(1)] (45)(1 - 0.7123) = 2.25.

In the risk neutral environment, the premium for a payment of 42 in one year is: $42e^{-.07}$.

Thus the premium for the given option is: $42e^{-.07} - 2.25 = 36.91$.

Therefore, the premium at time t of the given option is: $42e^{-.07(1-t)}$ - put premium.

Therefore, the delta of the given option is minus that of the put.

Put delta is: $-e^{-\delta T} N[-d_1]$ = - exp[-(0.03)(1)] (1 - 0.7123) = -0.2792.

Therefore, the Delta of the given option is: 0.2792.

$\Omega = \Delta S$ / (Option Premium) = (0.2792)(45)/36.91 = **0.34**.

<u>Comment</u>: The payoff on the given option is also: S - (payoff on a 42 strike call).

42. Prices for 6-month 60-strike European up-and-out call options on a stock S are available. Below is a table of option prices with respect to various H, the level of the barrier.
Here, S(0) = 50.

H	Price of up-and-out call
60	0
70	0.1294
80	0.7583
90	1.6616
∞	4.0861

Consider a special 6-month 60-strike European "knock-in, partial knock-out" call option that knocks in at H_1 = 70, and "partially" knocks out at H_2 = 80. The strike price of the option is 60.

The following table summarizes the payoff at the exercise date:

	H_1 Hit	
H_1 Not Hit	H_2 Not Hit	H_2 Hit
0	2 max[S(0.5) – 60, 0]	max[S(0.5) – 60, 0]

Calculate the price of the option.

(A) 0.6289 (B) 1.3872 (C) 2.1455 (D) 4.5856

(E) It cannot be determined from the information given above.

42. D. A barrier of infinity, means that the option is never knocked out.
Therefore, 4.0861 is the premium for a regular call.
knock-in plus knock-out = regular.
Thus a knock-in call with barrier 70 has a premium of: 4.0861 - 0.1294 = 3.9567.
A knock-in call with barrier 80 has a premium of: 4.0861 - 0.7583 = 3.3278.
Since the initial stock price is 50, if we hit 80 then we must have also hit 70.

Situation	Payoff Special Call	Payoff Knock-in H = 70	Payoff Knock-in H = 80
70 not hit	0	0	0
70 hit but not 80	$2(S - 60)_+$	$(S - 60)_+$	0
80 hit	$(S - 60)_+$	$(S - 60)_+$	$(S - 60)_+$

Therefore, the payoff on the special call is:
(2)(payoff on a knock-in call with barrier of 70) - (the payoff on a knock-in call with barrier of 80).
Therefore, the premium of the special call is: (2)(3.9567) - 3.3278 = **4.5856**.
Alternately, the payoff on the special call is equal to:
(payoff on an ordinary call) - (2)(payoff on a knock-out call with barrier of 70) +
(the payoff on a knock-out call with barrier of 80).

Situation	Special Call	Ordinary Call	Knock-out H = 70	Knock-out H = 80
70 not hit	0	$(S - 60)_+$	$(S - 60)_+$	$(S - 60)_+$
70 hit but not 80	$2(S - 60)_+$	$(S - 60)_+$	0	$(S - 60)_+$
80 hit	$(S - 60)_+$	$(S - 60)_+$	0	0

Therefore, the premium of the special call is: 4.0861 - (2)(0.1294) + 0.7583 = **4.5856**.
Comment: The current stock price is 50 and the strike price is 60. For the call to payoff, the stock price at expiration has to be greater than 60. Thus the stock price has to pass through 60. Therefore, if the barrier is 60, a knockout call is worthless.

43. Let $x(t)$ be the dollar/euro exchange rate at time t. That is, at time t, €1 = \$$x(t)$.
Let the constant r be the dollar-denominated continuously compounded risk-free interest rate.
Let the constant $r_€$ be the euro-denominated continuously compounded risk-free interest rate.

You are given

$$dx(t)/x(t) = (r - r_€)dt + \sigma\, dZ(t),$$

where $\{Z(t)\}$ is a standard Brownian motion and σ is a constant.
Let $y(t)$ be the euro/dollar exchange rate at time t. Thus, $y(t) = 1/x(t)$.
Which of the following equation is true?

(A) $dy(t)/y(t) = (r_€ - r)dt - \sigma\, dZ(t)$

(B) $dy(t)/y(t) = (r_€ - r)dt + \sigma\, dZ(t)$

(C) $dy(t)/y(t) = (r_€ - r - \sigma^2/2)dt - \sigma\, dZ(t)$

(D) $dy(t)/y(t) = (r_€ - r + \sigma^2/2)dt + \sigma\, dZ(t)$

(E) $dy(t)/y(t)= (r_€ - r + \sigma^2)dt - \sigma\, dZ(t)$

43. E. $dX = X(r - r_€)dt + X\sigma\, dZ$. $\Rightarrow dX^2 = X^2 \sigma^2 dt$.

$Y = 1/X$. $\partial Y/\partial X = -1/X^2$. $\partial^2 Y/\partial X^2 = 2/X^3$. $\partial Y/\partial t = 0$.

Using Ito's Lemma to get dY:

$dY = (-1/X^2)dX + (2/X^3)dX^2/2 = (-1/X^2)\{X(r - r_€)dt + X\sigma\, dZ\} + (1/X^3)X^2\sigma^2 dt =$

$(1/X)(r_€ - r + \sigma^2)dt - (1/X)\sigma\, dZ = Y(r_€ - r + \sigma^2)dt - Y\sigma\, dZ$.

$\mathbf{dY/Y = (r_€ - r + \sigma^2)dt - \sigma\, dZ}$.

Alternately, from the given equation X follows a Geometric Brownian Motion.

Therefore, $X(t) = X(0)\exp[(r - r_€ - \sigma^2/2)t + \sigma\, dZ(t)]$.

$Y(t) = 1/X(t) = \exp[(r_€ - r + \sigma^2/2)t - \sigma\, dZ(t)]/X(0) = Y(0)\exp[(r_€ - r + \sigma^2 - \sigma^2/2)t + \sigma(-dZ(t))]$.

Therefore, Y follows a Geometric Brownian Motion.

Thus, $\mathbf{dY/Y = (r_€ - r + \sigma^2)dt - \sigma\, dZ}$.

Comment: $dY = Y\{(r_€ - r_\$) + \sigma^2\}dt - Y\sigma\, dZ_\$$, where $dZ_\$$ is the dollar risk neutral process.

Y(t) is the number of Euros needed at time t to buy one dollar.

In the Euro risk neutral environment: $dY = Y(r_€ - r_\$)dt + Y\sigma\, dZ_€$, where $dZ_€$ is the euro risk neutral process

Note that the dollar risk neutral process is not equal to the Euro risk neutral process.

Let $dZ_€ = -dZ_\$ + \sigma\, dt$. $\Leftrightarrow dZ_\$ = -dZ_€ + \sigma\, dt$.

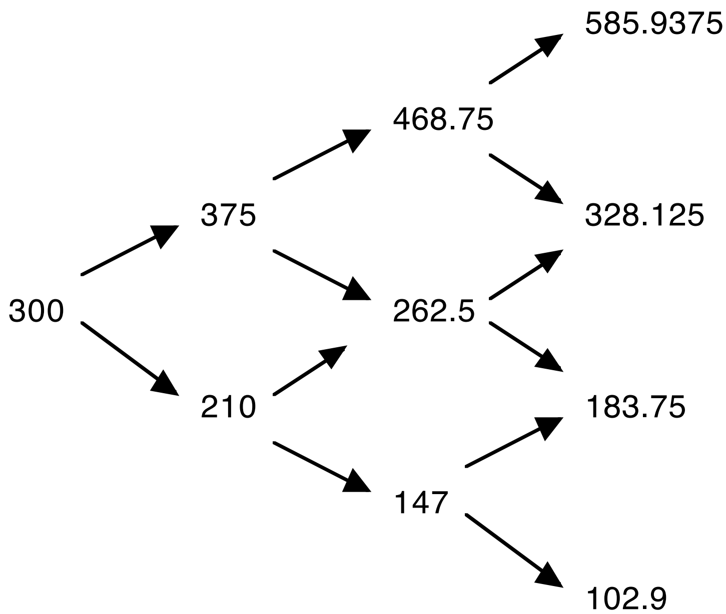
The minus sign on dZ comes from the fact that if the value of a dollar in Euros goes up, then the value of the Euro in dollars goes down, and vice-versa.

Then, $dY = Y\{(r_€ - r_\$) + \sigma^2\}dt - Y\sigma\, dZ_\$ = Y\{(r_€ - r_\$) + \sigma^2\}dt - Y\sigma(-dZ_€ + \sigma\, dt)$

$= Y(r_€ - r_\$)dt + Y\sigma\, dZ_€$.

This is closely related to Siegel's paradox: The forward exchange rate cannot be an unbiased estimate of the future spot exchange rate simultaneously for the dollar investor and the Euro investor. Usually the forward exchange rate is not an unbiased estimate of the future spot exchange rate in either environment. See Options, Futures, and Other Derivatives, by Hull.

For Questions **44** and **45**, consider the following three-period binomial tree model for a stock that pays dividends continuously at a rate proportional to its price. The length of each period is 1 year, the continuously compounded risk-free interest rate is 10%, and the continuous dividend yield on the stock is 6.5%.



44. Calculate the price of a 3-year at-the-money American put option on the stock.
(A) 15.86 (B) 27.40 (C) 32.60 (D) 39.73 (E) 57.49

45. Approximate the value of gamma at time 0 for the 3-year at-the-money American put on the stock using the method in Appendix 13.B of McDonald (2006).
(A) 0.0038 (B) 0.0041 (C) 0.0044 (D) 0.0047 (E) 0.0050

44. D. p* = (exp[(10% - 6.5%)(1)] - 210/300)/(375/300 - 210/300) = 61.0%.
At the 147 node, the continuation value is:

$\{(116.25)(61\%) + (197.10)(39\%)\}/e^{.1} = 133.72$.

The exercise value is: 300 - 147 = 153. Thus at this node we exercise early.

At the 262.5 node, the continuation value is: $(116.25)(39\%)/e^{.1} = 41.02$.

The exercise value is: 300 - 262.5 = 37.5. Thus at this node we do not exercise early.
At the 468.75 node, the continuation value and exercise value are both zero.
At the 210 node, the continuation value is:

$\{(41.02)(61\%) + (153)(39\%)\}/e^{.1} = 76.63$.

The exercise value is: 300 - 210 = 90. Thus at this node we exercise early.

At the 375 node, the continuation value is: $(41.02)(39\%)/e^{.1} = 14.48$.

The exercise value is zero. Thus at this node we do not exercise early.
At the initial node, the continuation value is:

$\{(14.48)(61\%) + (76.63)(39\%)\}/e^{.1} = 39.75$

The exercise value is zero. Thus at this node we do not exercise early.
The premium of the American put is **39.75**.
Comment: The value of the similar European put is:

$\{(.39^3)(197.1) + (3)(.39^2)(.61)(116.25)\}/e^{.3} = 32.63$.

45. C. $\Delta = e^{-\delta h} (P_u - P_d)/\{S_0(u - d)\}$.

At the 375 node, one period later the values of the put are 0 or 41.02.
At this node, Δ = exp[-6.5%](0 - 41.02)/(468,75 - 262.5) = -0.1864.
At the 210 node, one period later the values of the put are 41.02 or 153.
At this node, Δ = exp[-6.5%](41.02 - 153)/(262.5 - 147) = -0.9085.
$\Gamma \cong$ {-0.1864 - (-0.9085)}/(375 - 210) = **0.0044**.
Comment: In Appendix 13.B, McDonald applies this technique to a European option.

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months.

(ii) u/d = 4/3, where u is one plus the rate of gain on the futures price if it goes up,
and d is one plus the rate of loss if it goes down.

(iii) The risk-neutral probability of an up move is 1/3.

(iv) The initial futures price is 80.

(v) The continuously compounded risk-free interest rate is 5%.

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option. Determine C_{II} - C_I.

(A) 0 (B) 0.022 (C) 0.044 (D) 0.066 (E) 0.088

46. E. For Binomial Trees for options on futures contracts: $p^* = (1 - d)/(u - d)$.
Therefore, $1/3 = (1 - d)/(d4/3 - d)$. $\Rightarrow$ $d = 0.9$ and $u = 1.2$.
Therefore, the Binomial tree of forward prices is:

		115.2
	96	
80		86.4
	72	
		64.8

At the node where the price is 96, the exercise value is 96 - 85 = 11, and the continuation value is:
$\{(115.2 - 85)(1/3) + (86.4 - 85)(2/3)\}/e^{.05/2} = 10.7284$.
Therefore, we exercise the American call early at this node.
The difference in the call premiums is: $(1/3)(11 - 10.7284)/e^{.05/2}$ = **0.088**.
Comment: At page 332 of Derivative Markets by McDonald, $u = \exp[\sigma\sqrt{h}]$ and $d = \exp[-\sigma\sqrt{h}] = 1/u$. The u and d in this question were not gotten in this manner.
In the risk neutral environment, the expected forward price is equal to the current forward price:
$80 = u80(1/3) + d80(2/3)$. $\Rightarrow$ $1 = (4/3)d(1/3) + d(2/3)$. $\Rightarrow$ $d = 0.9$ and $u = 1.2$.
The premium of the European call is:
$\{(115.2 - 85)(1/3)^2 + (86.4 - 85)(2)(1/3)(2/3)\}/e^{.05} = 3.7838$.
At the node where the price is 72, the continuation value is:
$(86.4 - 85)(1/3)/e^{.05/2} = 0.4551$.
Therefore, the premium of the American call is:
$\{(1/3)(11) + (2/3)(0.4551)\}/e^{.05/2} = 3.8720$.

47. Several months ago, an investor sold 100 units of a one-year European call option on a nondividend-paying stock. She immediately delta-hedged the commitment with shares of the stock, but has not ever re-balanced her portfolio. She now decides to close out all positions.
You are given the following information:
(i) The risk-free interest rate is constant.
(ii)

	Several months ago	Now
Stock price	$40.00	$50.00
Call option price	$ 8.88	$14.42
Put option price	$ 1.63	$ 0.26
Call option delta	0.794	

The put option in the table above is a European option on the same stock and with the same strike price and expiration date as the call option.
Calculate her profit.
(A) $11 (B) $24 (C) $126 (D) $217 (E) $240

47. B. She buys 100 Δ = (100)(.794) = 79.4 shares of stock.
The cost to set up the position is: (79.4)(40) - (100)(8.88) = 2288.
The final value of the position is: (79.4)(50) - (100)(14.42) = 2528.
By Put-Call parity in the absence of dividends:

$8.88 + Ke^{-rT} = 40 + 1.63. \Rightarrow Ke^{-rT} = 32.75.$

$14.42 + Ke^{-r(T-t)} = 50 + 0.26. \Rightarrow Ke^{-r(T-t)} = 35.84.$

Dividing the two equations: $Ke^{rt} = 1.0943$.
Her profit is: (current value of her portfolio) - (accumulated value of her original investment) =
2528 - (2288)(1.0943) = **24.24**.
Alternately, she takes out a loan of 2288 and pays interest of (.0943)(2288) = 216.
Her profit is: 2528 - 2288 - 216 = **24**.

48. The prices of two nondividend-paying stocks are governed by the following stochastic differential equations:

$dS_1(t)/ S_1(t) = 0.06\ dt + 0.02\ dZ(t)$,

$dS_2(t)/ S_2(t) = 0.03\ dt + k\ dZ(t)$,

where Z(t) is a standard Brownian motion and k is a constant.
The current stock prices are $S_1(0) = 100$ and $S_2(0) = 50$.

The continuously compounded risk-free interest rate is 4%.
You now want to construct a zero-investment, risk-free portfolio with the two stocks and risk-free bonds.
If there is exactly one share of Stock 1 in the portfolio, determine the number of shares of Stock 2 that you are now to buy. (A negative number means shorting Stock 2.)

(A) -4 (B) -2 (C) -1 (D) 1 (E) 4

48. E. The two assets must have the same Sharpe Ratio.

(6% - 4%)/2% = (3% - 4%)/k. ⇒ k = -1%.

$dS_1 + xdS_2 = S_1(.06\ dt + .02\ dZ) + S_2x(.03\ dt - .01\ dZ) = 6\ dt + 2\ dZ) + x(1.5\ dt - 0.5\ dZ)$

$= (6 + 1.5x)dt + (2 - 0.5x)dZ$.

In order to be risk free, the coefficient multiplying dZ must be zero.

0 = 2 - 0.5x. ⇒ **x = 4**.

Comment: To buy the stocks it would cost: 100 + (4)(50) = 300. So you would have to borrow 300 by selling 300 in risk free bonds.
As on page 660 of Derivative Markets by McDonald,
the number of shares of stock 2 in the hedged portfolio is:

$-\sigma_1 S_1(0)/ \{\sigma_2 S_2(0)\} = - (2\%)(100)/\{(-1\%)(50)\} = 4$.

49. You use the usual method in McDonald and the following information to construct a one period binomial tree for modeling the price movements of a nondividend-paying stock.
(The tree is sometimes called a forward tree).
(i) The period is 3 months.
(ii) The initial stock price is $100.
(iii)The stock's volatility is 30%.
(iv) The continuously compounded risk-free interest rate is 4%.
At the beginning of the period, an investor owns an American put option on the stock.
The option expires at the end of the period.
Determine the smallest integer-valued strike price for which an investor will exercise the put option at the beginning of the period.
(A) 114 (B) 115 (C) 116 (D) 117 (E) 118

49. B. u = exp[4%/4 + 30/√4] = 1.1735. d = exp[4%/4 - 30/√4] = 0.8694.
p* = (exp[4%/4] - 0.8694)/(1.1735 - 0.8694) = 46.25%.
S_0 = 100, S_u = 117.35, and S_d = 86.94.

Assuming K ≤ 117.35, then the continuation value is: (1 - .4625)(K - 86.94)/$e^{.01}$.
At time zero, the exercise value would be greater than the continuation value if:
K - 1000 > (1 - .4625)(K - 86.94)/$e^{.01}$. ⇒ K > 114.85.
So if K = **115** we would exercise early.
Comment: You can try the choices, starting with C.
If K = 116, we would exercise early, so then we check K = 115.
We would also exercise early if K = 115 , so then we check K = 114.
We would not exercise early if K = 114.

K	Continuation Value	Exercise Value	Exercise Early?
114	14.40	14	No
115	14.93	15	Yes
116	15.46	16	Yes
117	16.00	17	Yes
118	16.83	18	Yes

If K = 118, the continuation value is:
{(.4625)(118 - 117.35) + (1 - .4625)(118 - 86.94)}/$e^{.01}$ = 16.83.