Section 6, Options on Currency

For example, one may be able today to buy one Euro for 1.25 U.S. Dollars.
1€ = \$1.25. ⇔ 0.8€ = \$1. Instead one could buy one U.S. Dollar for 0.8 Euros.

Let $r_\$$ be the interest rate on dollars and $r_€$ be the interest rate on Euros.
Then the forward price in dollars for one Euro in the future is: (current exchange rate)exp[($r_\$$ - $r_€$)T].[98]
The forward price in Euros for one dollar in the future is: (current exchange rate)exp[($r_€$ - $r_\$$)T].[99]

In general, one can exchange one currency for another currency at an exchange rate.[100]
However, exchange rates change over time. Therefore, it is useful for businesses engaged in international trade to buy and sell options related to currencies.

Dollar Denominated Options:

For example, one could purchase a call option to buy one euro in 2 years for \$1.20.
The owner of this call would have the option in 2 years to use \$1.20 to obtain one Euro.

This is an example of a dollar-denominated option, since the strike price and premium (cost) are in dollars. A dollar-denominated call on Euros would give one the option to obtain Euros at some time in the future for a specified number of dollars.

If the price of one euro in two years is more than \$1.20, then one would exercise the above option. For example if the exchange rate two years from now is \$1.50 per Euro, then one would make \$1.50 - \$1.20 = \$0.30 by exercising this call.

A dollar-denominated put on Euros would give one the option to sell Euros at some time in the future for a specified number of dollars. For example, a 2 year \$1.20 strike put on Euros would give its owner the option in 2 years to sell one Euro for \$1.20. The owner would exercise this put if 2 years from now the exchange rate is less than \$1.20 per Euro.

The payoff of the call option is: $(x_2 - \$1.20)_+$, where x_2 is the exchange rate in two years.
The payoff of the put option is: $(\$1.20 - x_2)_+$, where x_2 is the exchange rate in two years.

The payoff on a call on currency has the same mathematical expression as for a call on stocks, with x_T substituted for S_T: $(x_T - K)_+$. Similarly, the payoff on a put on currency is: $(K - x_T)_+$.

[98] See Equation 5.18 in Derivative Markets by McDonald, on the syllabus of an earlier exam.
[99] Since the current exchange rates are reciprocals of each other, so are the forward prices.
[100] For current exchange rates see for example www.xe.com/ucc/

Put-Call Parity for Dollar Denominated Options:[101]

The same put-call parity relationship holds as was discussed previously:
$C(K, T) = P(K, T) + PV[F_{0,T}] - PV[K]$.

If the option is dollar-denominated, then the strike price K is in dollars. We discount it back to the present using the interest rate for dollars $r_\$$, usually just written as r in the United States.

The present value of one euro at time T, $PV[F_{0,T}]$, is the amount we would need to invest now in a euro-denominated risk free bond in order to have 1 euro at time t. A euro-denominated risk free bond earns interest continuous at a rate of $r_€$.[102]

If we spend $x_0 \exp[-Tr_€]$ dollars to buy $\exp[-Tr_€]$ euros at time 0, and invest in a euro-denominated risk free bond, we will have 1 euro at time T.
Therefore, $PV[F_{0,T}] = x_0 \exp[-Tr_€]$.
Therefore, for dollar-denominated options on euros:

$$\mathbf{C_\$(x_0, K, T) = P_\$(x_0, K, T) + x_0 \exp[-Tr_€] - K e^{-Tr}}.$$ [103]

x_0 is the exchange rate for Euros in terms of dollars, \$1.25 = 1 Euro, and K is in dollars.

This is analogous to what we had for options on stock: $C(K, T) = P(K, T) + S_0 e^{-T\delta} - K e^{-Tr}$.

$\mathbf{x_0 \Leftrightarrow S_0}$.

Dollars act as money: $\mathbf{r_\$ \Leftrightarrow r}$.

Euros act as the asset: $\mathbf{r_€ \Leftrightarrow \delta}$.

Exercise: x_0 = \$1.25/€ . T = 2 years. K = \$1.20/€ .
The dollar-denominated interest rate is 6% and the euro-denominated interest rate is 4%.
The option premium for a dollar-denominated put on one euro is \$0.10.
Determine the premium for a dollar-denominated call on one euro.
[Solution: $\$0.10 + (\$1.25)e^{-(2)(.04)} - (\$1.20)e^{-(2)(.06)} = \$0.190$.
Comment: Note how every term in the equation is in dollars. This is one way to check your work.]

[101] See page 286 of Derivative Markets by McDonald.
[102] We assume that $r_\$$ will usually differ from $r_€$. Similarly, each currency will have its own associated interest rate.
[103] See equation 9.4 of Derivative Markets by McDonald.
This is similar to the parity equation for stocks paying continuous dividends, with $r_€$ taking the place of δ.

Foreign Denominated Options:

A Euro-denominated call on dollars would give one the option to obtain dollars at some time in the future for a specified number of Euros.

For example, a 2 year € 0.70 strike call on dollars would give its owner the option in 2 years to buy one dollar for € 0.70. The owner would exercise this call if 2 years from now the exchange rate is more than € 0.70 per dollar.

A Euro-denominated put on dollars would give one the option to sell dollars at some time in the future for a specified number of Euros.

For example, a 2 year € 0.60 strike put on dollars would give its owner the option in 2 years to sell one dollar for € 0.60. The owner would exercise this put if 2 years from now the exchange rate is less than € 0.60 per dollar.

Put-Call Parity for Foreign Denominated Options:

For Euro-denominated options on dollars:

$$\mathbf{C_{€}(x_0, K, T) = P_{€}(x_0, K, T) + x_0 \exp[-T\, r_{\$}] - K \exp[-T\, r_{€}]}.$$

x_0 is the exchange rate for dollars in terms of Euros, 0.8 Euro = $1, and K is in Euros.

This is analogous to what we had for options on stock: $C(K, T) = P(K, T) + S_0\, e^{-T\delta} - K\, e^{-Tr}$.

$\mathbf{x_0 \Leftrightarrow S_0}$.

Euros act as money: $\mathbf{r_{€} \Leftrightarrow r}$.

Dollars act as the asset: $\mathbf{r_{\$} \Leftrightarrow \delta}$.

Exercise: x_0 = 0.8€ /$. T = 2 years. K = 0.833€ /$.
The dollar-denominated interest rate is 6% and the euro-denominated interest rate is 4%.
The option premium for a euro-denominated put on one dollar is 0.08 euro.
Determine the premium for a euro-denominated call on one dollar.
[Solution: 0.08 euros + (.8 euros)$e^{-(2)(.06)}$ - (.833 euros)$e^{-(2)(.04)}$ = 0.0206 euro.
Comment: Since the options are euro-denominated, the roles of the two interest rates are reversed. Here euros act as the currency, while dollars act as the asset. Therefore, the interest rate on dollars is analogous to the dividend rate on a stock.
Note how every term in the equation is in euros. This is one way to check your work.]

Different Points of View:

Assume that currently 1€ = \$1.25. ⇔ 0.8€ = \$1.

Sam is in the United States. Sam has 1000 dollar denominated calls with strike price \$1.30, giving him the option one year from now to buy 1000 Euros for 1300 dollars.

Exercise: In one year, 1€ = \$1.4. What is the payoff on Sam's calls?
[Solution: \$1.4 - \$1.30 = \$0.10. For 1000 calls: (1000)(\$0.10) = \$100.]

From Sam's point of view, the future value of each of his calls in dollars is: $(x_1 - 1.30)_+$.

Johann is in Germany. Johann has 1300 Euro denominated puts with strike price: 1/1.30 = .7692, giving him the option one year from now to sell 1300 dollars for: (1300)(.7692) = 1000 Euros.

Exercise: In one year, 1€ = \$1.4. What is the payoff on Johann's puts?
[Solution: \$1 = 1/1.4 = 0.7143 euros. .7692 - 0.7143 = 0.0549 euros.
(1300)(0.0549) = 71.4 euros.
Comment: At this future exchange rate, 71.4 euros = (71.4€)(\$1.4/€) = \$100.]

From Johann's point of view, the payoff on each of his puts in euros is: $(.7692 - 1/x_1)_+$,
where x_1 is the exchange rate to buy euros with dollars, and therefore $1/x_1$ is the exchange rate to buy dollars with euros. The payoff in dollars on each of Johann's puts is:
$x_1(.7692 - 1/x_1)_+ = (.7692x_1 - 1)_+ = (x_1 - 1.30)_+/1.30$.

Sam's calls have value if $x_1 > 1.30$. Johann's puts have value if $.7682 > 1/x_1 \Leftrightarrow x_1 > 1.30$.
Their options have value under the same conditions.

If $x_1 = 1.4$, then as calculated above, Sam's calls are worth \$100 and Johann's puts are worth 71.4 euros or \$100 at this future exchange rate. Johann's puts are worth the same as Sam's calls.

Both Sam's and Johann's positions give them the option one year from now to use 1300 dollars to obtain 1000 Euros. Therefore, their positions must be worth the same amount.

Exercise: If each of Johann's euro-denominated puts for one dollar with a strike price of 0.7692 euros has a price of 0.0523 euros, determine the price of Sam's dollar denominated one year call for one euro, with a strike price of $1.30.
[Solution: Johann's position costs (1300)(0.0523) = 68 euros.
Thus each of Sam's calls should cost: 68 euros/1000 = 0.068 euros.
At the current exchange rate, this is: (0.068 euros)($1.25/ euro) = $0.085.
Comment: Sam's position is worth: (1000)($0.085) = $85.]

In general, in order to replicate the dollar denominated call, one needs to buy K euro denominated puts. Thus the value of the call is K times the value of the put. In order to put the price of the call in dollars rather than euros, one must multiply by the current exchange rate x_0.

$\mathbf{C_{\$}(x_0, K, T) = x_0\ K\ P_f(1/x_0, 1/K, T)}$.[104]

Exercise: Use the above formula to solve the previous exercise.
[Solution: $C_{\$}(1.25, 1.3, T) = (1.25)\ (1.3)\ P_{€}(1/1.25, 1/1.3, T) = (1.25)(1.3)(0.0523) =$ $0.085.]

Calls from Sam's point of view was equivalent to puts from Johann's point of view.
A call from one point of view is a put from an opposite point of view.

Similarly, $\mathbf{P_{\$}(x_0, K, T) = x_0\ K\ C_f(1/x_0, 1/K, T)}$.

For a dollar denominated call, we have the option to use dollars to receive another currency.
For a dollar denominated put, we have the option to receive dollars by using another currency.

For a Euro denominated call, we have the option to use Euros to receive another currency.
For a Euro denominated put, we have the option to receive Euros by using another currency.

[104] See equation 9.7 in Derivative Markets by McDonald.

Derivation of the Relationship of Dollar and Foreign Denominated Currency Options:

Let x(t) be the exchange rate at time t; x(t) is the value in dollars of one of the foreign currency at time t.[105]

The payoff at expiration of a K-strike dollar denominated call on a foreign currency is in dollars:
$(x(T) - K)_+$.

Now at time T, this payoff would be equal to in the foreign currency:
$(x(T) - K)_+/x(T) = (1 - K/x(T))_+ = K(1/K - 1/x(T))_+$.

The payoff at expiration of a 1/K-strike foreign denominated put on dollars is in that foreign currency:
$(1/K - 1/x(T))_+$.

Therefore, both in the foreign currency at expiration:
K(payoff of 1/K-strike foreign denominated put on dollars) =
(payoff of K-strike dollar denominated call on foreign currency).

Therefore, K(premium of 1/K-strike foreign denominated put on dollars) =
(premium in foreign currency of K-strike dollar denominated call on foreign currency) =
(premium in dollars of K-strike dollar denominated call on foreign currency) / x_0.[106]

In other words, $C_\$(x_0, K, T) / x_0 = K\ P_f(1/x_0, 1/K, T)$.

$\Rightarrow C_\$(x_0, K, T) = x_0\ K\ P_f(1/x_0, 1/K, T)$.

[105] For example, the current exchange rate could be \$0.8 per Euro.
[106] For example, if one could pay 0.10 Euros to purchase the dollar denominated call on Euros, and the current exchange rate were \$0.8 per Euro, then one could also pay 0.10/0.8 = \$0.125 to purchase this call.

Examples of Using Currency Options for Hedging:

Aidan Math is an American consulting actuary. On January 1, Aidan takes a three month assignment consulting for the Seguros Popular, a Mexican Insurance Company. Aidan will be paid a lump sum for his work on March 31 at the completion of his assignment. Aidan will be doing all the work at his office in the United States.

The exchange rate on January 1 is 10 pesos per U.S. dollar, or $0.10 U.S. per Mexican Peso.

If Seguros Popular agrees to pay Aidan 600,000 Pesos on March 31, then Aidan faces a risk of the exchange rate changing. On January 1, 600,000 Pesos is equivalent to $60,000. If on March 31 the exchange rate were 12 pesos per U.S. dollar, then Aidan would receive only the equivalent of 600,000/12 = $50,000.

Aidan would have the choice of just accepting this risk, or buying an option to hedge this risk. Aidan could buy 60,000 peso denominated 3 month 10 peso European call options to buy dollars.[107] [108] Then on March 31, if the exchange rate is more than 10 peso per dollar, he can exercise his options and use the 600,000 Pesos he is paid by Seguros Popular to buy $60,000.

If instead on March 31 the exchange rate were 8 pesos per U.S. dollar, then Aidan would receive the equivalent of 600,000/8 = $75,000. In this case, Aidan would benefit from the movement in the exchange rate. If on March 31 the exchange rate were less than 10 pesos per U.S. dollar, then the 600,000 pesos Aidan receives would be the equivalent of more than $60,000.

Instead of accepting this possibility of being lucky, Aidan could have sold 600,000 peso denominated 3 month 10 peso European put options to buy dollars.[109] In that case, if the exchange rate were less than 10 pesos per U.S. dollar, then the person who bought these puts from Aidan would exercise them, and buy the 600,000 Pesos that Aidan receives from Seguros Popular for $60,000.[110] Aidan would no longer benefit if the exchange rate were less than 10 pesos per U.S. dollar, but in any case he would have the money from selling the puts. This money would help to offset the cost of the calls he bought.

[107] Equivalently, Aidan could buy 600,000 dollar denominated 3 month $0.1 European put options to sell pesos. Then on March 31, if the exchange rate is more than 10 peso per dollar, in other words less than $0.1 per peso, he can exercise his option and sell the 600,000 Pesos he is paid by Seguros Popular for $60,000.

[108] As will be discussed subsequently, such a call could be priced using the Black-Scholes formula. If σ = 12%, the interest rate in pesos is 6%, and the interest rate in dollars is 5%, then the premium of each such call would be about 0.25 pesos. Thus 60,000 such calls would cost Aidan about 15,000 pesos on January 1, the equivalent of $1500.

[109] Each such put would cost about 0.224 pesos, and Aidan would receive about $1460. Thus if Aidan both buys the calls and sells the puts, then for a net cost of about $1500 - $1460 = $40, he would be unaffected by changes in currency exchange rates. In this case, the calls cost more than the puts, since I have assumed the interest rate in pesos is greater than the interest rate in dollars.

[110] For example, if the exchange rate on March 31 were 8 pesos per U.S. dollar, obtaining 600,000 pesos would normally cost 600,000/8 = $75,000. Using the options bought from Aidan, this person could instead obtain 600,000 pesos for only $60,000.

Since Aidan will be doing all the work at his office in the United States, he would prefer to be paid in dollars. If Seguros Popular agrees to pay Aidan $60,000 on March 31, then Seguros Popular rather than Aidan would face a risk of the exchange rate changing. In this case, if on March 31 the exchange rate were 12 pesos per U.S. dollar, then Seguros Popular would have to use: (12)(60000) = 720,000 pesos in order to pay $60,000 to Aidan.

Similar to the previous situation, Seguros Popular would have the choice of just accepting this risk, or buying an option to hedge this risk. Seguros Popular could buy 60,000 peso denominated 3 month 10 peso European call options to buy dollars.[111] Then on March 31, if the exchange rate is more than 10 peso per dollar, Seguros Popular can exercise its options and use the 600,000 Pesos to buy $60,000, with which to pay Aidan. If on March 31 the exchange rate is less than 10 peso per dollar, then Seguros Popular would benefit from the exchange rate movement; they would need to use less than 600,000 pesos in order to pay Aidan $60,000.

Rosetta Stone is a Mexican pension actuary. On January 1, Rosetta takes a six month assignment consulting for the Grate American Cheese Company, which is based in the United States. Rosetta will be paid a lump sum for her work on June 30 at the completion of her assignment. Rosetta will be doing all the work at her office in Mexico.

The exchange rate on January 1 is 10 pesos per U.S. dollar, or $0.10 U.S. per Mexican Peso.

Exercise: Grate American Cheese Company has agreed to pay Rosetta $100,000. Briefly discuss the currency exchange risk faced by Rosetta and how she can hedge it using options.
[Solution: If the exchange rate is less than 10 pesos per dollar, then the $100,000 Rosetta will be paid is worth less than the 1 million pesos she expected. Rosetta could buy 100,000 peso denominated 6 month 10 peso European put options to sell dollars. Alternately, Rosetta could buy 1 million dollar denominated 6 month $0.1 European call options to buy pesos.]

Exercise: Grate American Cheese Company has agreed to pay Rosetta 1 million pesos. Briefly discuss the currency exchange risk faced by the company and how to hedge it using options.
[Solution: If the exchange rate is less than 10 pesos per dollar, then the 1 million pesos Rosetta will be paid will cost the company more than the $100,000 it expected. The company could buy 1 million dollar denominated 6 month $0.1 peso European call options to buy pesos. Alternately, it could buy 100,000 peso denominated 6 month 10 peso European put options to sell dollars.
<u>Comment</u>: In this and the previous exercise, Rosetta and the company each face the same risk, and can hedge that risk the same way. If the company had agreed to pay Rosetta $50,000 and 500,000 pesos, then Rosetta and the company would share the currency exchange risk.]

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[111] Equivalently, Seguros Popular could buy 600,000 dollar denominated 3 month $0.1 European put options to sell pesos.

Problems:

6.1 (2 points) The spot exchange rate of dollars per euro is 1.1.
Dollar and euro interest rates are 5.0% and 6.0%, respectively.
The price of a $0.93 strike 18-month call option is $0.28.
What is the price of a similar put?
A. Less than $0.05
B. At least $0.05, but less than $0.10
C. At least $0.10, but less than $0.15
D. At least $0.15, but less than $0.20
E. At least $0.20

6.2 (2 points) Currently one can buy one dollar for 120 yen.
Dollar and yen interest rates are 4.5% and 3.0%, respectively.
The price of a 100 yen strike 2 year put option is 13 yen.
What is the price in yen of a similar call?
A. Less than 15
B. At least 15, but less than 20
C. At least 20, but less than 25
D. At least 25, but less than 30
E. At least 30

6.3 (2 points) Currently one can buy one Swiss Franc for 0.40 British Pounds (£0.4).
The interest rate for Swiss Francs is 5%.
The interest rate for British Pounds is 3%.
The price of a £0.5 strike 3 year call option is £0.05.
Determine the price in Swiss Francs of a 2 Swiss Franc strike 3 year put option.
A. 0.04 B. 0.05 C. 0.10 D. 0.125 E. 0.25

6.4 (2 points) The price of a $0.02 strike 1 year call option on an Indian Rupee is $0.00565.
The price of a $0.02 strike 1 year put option on an Indian Rupee is $0.00342.
Dollar and rupee interest rates are 4.0% and 7.0%, respectively.
How many dollars does it currently take to purchase one rupee?
A. 0.020 B. 0.021 C. 0.022 D. 0.023 E. 0.24

6.5 (2 points) The current exchange rate is $1.90 per British Pound.
A $2 strike 6 month call on the British Pound currently costs $0.0554.
What is the price in pounds of a £0.5 strike 6 month put on United States dollars?
A. £0.010 B. £0.015 C. £0.020 D. £0.025 E. £0.030

6.6 (2 points) Currently one can buy 1 Brazilian Real for $0.47.
Dollar and Real interest rates are 4.0% and 7.0%, respectively.
The price of a $0.50 strike 6-month call option to buy 1 Brazilian Real is $0.09.
What is the price of a similar put?
A. Less than $0.05
B. At least $0.05, but less than $0.10
C. At least $0.10, but less than $0.15
D. At least $0.15, but less than $0.20
E. At least $0.20

6.7 (2 points) The current exchange rate is $0.15 per Chinese Yuan Renminbi.
An 8 yuan strike 2 year call on the U.S. Dollar currently costs 0.568 yuan.
What is the price in dollars of a $0.125 strike 2 year put on yuan?
A. Less than $0.006
B. At least $0.006, but less than $0.008
C. At least $0.008, but less than $0.010
D. At least $0.010, but less than $0.012
E. At least $0.012

6.8 (2 points) Currently one can buy 1 U.S. Dollar for 11 Mexican Pesos.
Dollar and Peso interest rates are 4.0% and 6.0%, respectively.
The price of a 12 Peso strike 1-year put option is 1.36 Pesos.
What is the price in Pesos of a similar call?
A. Less than 0.60
B. At least 0.60, but less than 0.70
C. At least 0.70, but less than 0.80
D. At least 0.80, but less than 0.90
E. At least 0.90

6.9 (3 points) The spot exchange rate of dollars per South Korean Won is 0.00097.
Dollar and Won interest rates are 4.0% and 7.0%, respectively.
The price for one million $0.00100 strike 3 year call options on Won is $40.11.
What is the price in Won of a 1000 Won strike 3 year call on dollars?
A. 135 B. 140 C. 145 D. 150 E. 155

6.10 (3 points) The spot exchange rate of yen per Euro is 125.
You are given the following premiums for five year Euro denominated calls on yen.

Strike	Premium
□0.004	□0.003618
□0.006	□0.002132
□0.008	□0.001072
□0.010	□0.000482
□0.012	□0.000203
□0.014	□0.000083

Determine the premium for a five year 100 yen strike put on Euros.
A. 5 B. 6 C. 9 D. 13 E. 17

6.11 (3 points) You are given:
(i) The current exchange rate is 1.16€/£.
(ii) A two-year European call option on Euros with a strike price of £0.80 sells for £0.120.
(iii) The continuously compounded risk-free interest rate on British Pounds is 5%.
(iv) The continuously compounded risk-free interest rate on Euros is 3%.
Calculate the price of 1000 two-year European call options on British Pounds with a strike price of €1.25.
(A) €44 (B) €46 (C) €48 (D) €50 (E) €52

6.12 (5B, 5/99, Q.14) (1 point) An American manufacturer has contracted to sell a large order of widgets for one million Canadian dollars, with payment due on delivery in six months.
How could the American company reduce foreign exchange risk?
1. Borrow Canadian currency against its receivables, convert to U.S. dollars at the spot rate, and invest proceeds in the U.S.
2. Buy an option to sell Canadian dollars in six months at a specific price.
3. Buy Canadian currency forward.
A. 1 B. 1, 2 C. 1, 3 D. 2, 3 E. 1, 2, 3

6.13 (CAS3, 11/07, Q.15) (2.5 points)
A nine-month dollar-denominated call option on euros with a strike price of $1.30 is valued at $0.06.
A nine-month dollar-denominated put option on euros with the same strike price is valued at 0.18.
The current exchange rate is $1.2/euro and the continuously compounded risk-free rate on dollars is 7%. What is the continuously compounded risk-free rate on euros?
A. Less than 7.5%
B. At least 7.5%, but less than 8.5%
C. At least 8.5%, but less than 9.5%
D. At least 9.5%, but less than 10.5%
E. At least 10.5%

6.14 (MFE/3F, 5/09, Q.9) (2.5 points) You are given:
(i) The current exchange rate is 0.011$/¥.
(ii) A four-year dollar-denominated European put option on yen with a strike price of $0.008 sells for $0.0005.
(iii) The continuously compounded risk-free interest rate on dollars is 3%.
(iv) The continuously compounded risk-free interest rate on yen is 1.5%.
Calculate the price of a four-year yen-denominated European put option on dollars with a strike price of ¥125.
(A) ¥35 (B) ¥37 (C) ¥39 (D) ¥41 (E) ¥43

Section 7, Exchange Options[112]

One can buy an option to exchange one asset for another.

For example, Steve is given the option to exchange one share of a competitor's Stock B for one share of the stock of his employer Stock A. This is a call option with underlying asset Stock A and strike asset Stock B.

A call exchange option allows one to exchange one share of a Stock B (strike asset) for one share of the stock of Stock A (underlying asset). So rather than a strike price K in dollars, we have the option to use an asset, Stock B, in order to obtain Stock A.

Assuming a European call, then at time T, Steve would make the exchange if a share of Stock A is worth more than a share of Stock B. The future value of the call is $(S_T - Q_T)_+$, where S is price of the underlying asset, Stock A, and Q is the price of the strike asset, Stock B.

Whether one will exercise the call depends on the relative value of the underlying asset and the strike asset, both of which are random variables. If at expiration the underlying asset is worth more than the strike asset, then it would be worthwhile to exercise the call.

For example, if the call expires in 2 years, among the many possible situations:

Price of Stock A (Underlying Asset) 2 Years from now	Price of Stock B (Strike Asset) 2 Years from now	Exercise Exchange Call?
100	90	Yes
110	130	No
120	100	Yes
70	80	No

A similar put exchange option would instead allow us to sell stock A in exchange for Stock B, in other words, get B in exchange for A.

[112] See Section 9.2 of Derivative Markets by McDonald. Also called an outperformance option.

Put-Call Parity:

Buy one European call, sell a similar put, sell a prepaid forward contract on Stock A, and buy a prepaid forward contract on Stock B.[113]

At Time T, you will get a share of Stock B due to the forward contract you bought.

If $S_T \geq Q_T$, you will exercise your call and exchange your Share of Stock B for a Share of Stock A. Then you will provide this share of Stock A to the person who bought a forward contract from you.

If instead $S_T < Q_T$, then the person who bought your put, will exchange a share of Stock A for your share of Stock B. You will then provide this share of Stock A to the person who bought a forward contract from you.

In either case, at time T you end up with nothing after all of the transactions.
Therefore, the correct price for this position is zero.

Let $F_{0,T}^P(S_0)$ be the prepaid forward price of a share of Stock A.

Let $F_{0,T}^P(Q_0)$ be the prepaid forward price of a share of Stock B.

We have shown that: $0 = C_{Eur}(S_0, Q_0, T) - P_{Eur}(S_0, Q_0, T) - F_{0,T}^P(S_0) + F_{0,T}^P(Q_0)$.

$$\mathbf{C_{Eur}(S_0, Q_0, T) = P_{Eur}(S_0, Q_0, T) + F_{0,T}^P(S_0) - F_{0,T}^P(Q_0)}.$$ [114] [115]

If the current price of Stock A is \$100 and the current price of Stock B is \$120, and neither one pays dividends, then $F_{0,T}^P(S_0) = 100$ and $F_{0,T}^P(Q_0) = 120$. Therefore,

$C_{Eur}(S_0, Q_0, T) = P_{Eur}(S_0, Q_0, T) + 100 - 120 = P_{Eur}(S_0, Q_0, T) - 20$.

If instead Stock A pays dividends at a continuous rate δ_S and Stock B pays dividends at a continuous rate δ_Q, then $F_{0,T}^P(S_0) = S_0 \exp[-T\delta_S]$ and $F_{0,T}^P(Q_0) = Q_0 \exp[-T\delta_Q]$.[116] Therefore:

$$C_{Eur}(S_0, Q_0, T) = P_{Eur}(S_0, Q_0, T) + S_0 \exp[-T\delta_S] - Q_0 \exp[-T\delta_Q].$$

[113] Under a prepaid forward contract, you pay now but receive the asset at a fixed point of time in the future.
[114] See Equation 9.6 in Derivative Markets by McDonald.
[115] This put-call parity follows from the fact that: $(S(T) - Q(T))_+ + Q(T) = (Q(T) - S(T))_+ + S(T)$.
See remark (iii) on MFE Sample Exam Q.55.
[116] While you pay for the prepaid forward contract now, you do not get the dividends that are paid on the stock between now and time T, since you do not own the stock until time T.

Exercise: The current price of Stock A is \$100 and the current price of Stock B is \$120.
Stock A pays dividends at a continuous annual rate of 1%, while Stock B pays dividends at a continuous annual rate of 2%. What is the difference in price between a 3 year European call option to exchange Stock B for Stock A, and the similar put?
[Solution: $C_{Eur}(S_0, Q_0, 3) - P_{Eur}(S_0, Q_0, 3) = (100)\exp[-(3)(1\%)] - (120)\exp[-(3)(2\%)] = -15.97$.
Comment: In this case, the put is worth more than the similar call. Exchange Call: can use B to get A. Exchange Put: can use A to get B. Since Stock B is currently worth much more than Stock A, the put is more likely to be worth something at expiration than is the call.]

$C_{Eur}(S_0, Q_0, T) = P_{Eur}(S_0, Q_0, T) + F_{0,T}^P(S_0) - F_{0,T}^P(Q_0)$.

Therefore, the premium of the put is equal to the premium of the similar call, if and only if the two stocks have the same prepaid forward price.

Note that this form of put-call parity includes the situations previously discussed as special cases. For example, if the strike asset is money, specifically K dollars, then the "dividend" on money is interest at rate r, and

$C_{Eur}(S_0, K, T) = P_{Eur}(S_0, K, T) + S_0 \exp[-T\delta_S] - K \exp[-Tr]$.

"**Parity is the observation that buying a European call and selling a European put with the same strike price and time to expiration is equivalent ot making a leveraged investment in the underlying asset, less the value of cash payments to the underlying asset over the life of the option.**"[117]

[117] See page 305 of Derivatives Markets by McDonald.

Points of View:

As discussed previously, a call from one point of view is a put from another point of view.

Neither Stock A nor Stock B pay dividends.
Adam has a 2 year European call with underlying asset Stock A and strike asset Stock B.
Adam has the option 2 years from now to buy a share of Stock A in exchange for a share of Stock B. He will do so if two years from now Stock A is worth more than Stock B.

Eve has a 2 year European put with underlying asset Stock B and strike asset Stock A.
Eve has the option 2 years from now to sell a share of Stock B in exchange for a share of Stock A . She will do so if two years from now Stock A is worth more than Stock B.

Two years from now they will each own whichever of the two stocks is worth more.
Adam and Eve have options with the same outcome and therefore the same value.

In the absence of dividends, $C(S_0, Q_0, T) = P(Q_0, S_0, T)$.

Call to buy Stock A for price Stock B: A is the underlying asset and B is the strike asset.

Put to sell Stock B for price Stock A: B is the underlying asset and A is the strike asset.

Despite the different language, both give you the option to use Stock B to obtain Stock A.

Problems:

7.1 (2 points) Consider the case of a European exchange option in which the underlying stock is Widget Co. with a current price of $65 per share. The strike asset is Gadget Inc., with a per share price of $62. Neither stock will pay dividends within the next 3 months. Interest rates are 5% and the 3 month call option is trading for $8. What is the price of the similar put?
A. 3 B. 5 C. 7 D. 9 E. 11

7.2 (2 points) A 2 year European exchange call option has underlying asset one share of MGH Shipping with a current price of $100 per share. The strike asset is 2 shares of Galactus Transport, with a per share price of $60. One will have the option to use 2 shares of Galactus in order to obtain one share of MGH. The price of this call option is $11.
MGH Shipping pays dividends at a continuous rate of 2% per year.
Galactus Transport pays dividends at a continuous rate of 1% per year.
What is the price of the similar put?
A. Less than $20
B. At least $20, but less than $25
C. At least $25, but less than $30
D. At least $30, but less than $35
E. At least $35

7.3 (2 points) A 3 year European exchange option has underlying asset one share of Peach Computer Company with a current price of $200 per share. The strike asset is the Silicon Valley Stock Index, with a current price of $210. The price of this call option is $13.
Peach Computer Company pays dividends at a continuous rate of 3% per year.
Silicon Valley Stock Index pays dividends at a continuous rate of 1% per year.
What is the price of the similar put?
A. Less than $20
B. At least $20, but less than $25
C. At least $25, but less than $30
D. At least $30, but less than $35
E. At least $35

7.4 (2 points) Consider four-year European exchange options involving the stocks of Global Dynamics and Deon International. The price of an option to exchange a share of Global Dynamics for a share of Deon International is the same as the price of an option to exchange a share of Deon International for a share of Global Dynamics.
Global Dynamics has a current price of 92 and pays dividends at a continuous rate of 2% per year.
Deon International pays dividends at a continuous rate of 0.9% per year.
What is the current price of a share of Deon International?
A. 85 B. 86 C. 87 D. 88 E. 89

Section 8, Futures Contracts[118]

As discussed previously, **a forward contract is an agreement that sets the terms today, including price and quantity, at which one will buy or sell an asset or commodity at a specific time in the future. A futures contract is similar, except that the buyer and seller post margin, the contract is marked to market, and the contract is typically traded on an exchange**.[119]

An Example of a Futures Contract:

Frank is a farmer. He expects to harvest 10,000 bushels of wheat in July. On April 15, Frank enters into a futures contract for $4 per bushel agreeing to deliver 10,000 bushels of wheat on July 15 to Mom's Bakery.[120] Frank has sold a futures contract.[121] Frank's position would be referred to as the short position; he has agreed to deliver wheat. Mom's Bakery has bought a futures contract. Mom's Bakery has a long position; it has agreed to accept delivery of wheat at a predetermined price.[122]

Frank has guarded against the risk of the price of wheat falling between April and July. Note that Frank has not guarded against for example his crop being destroyed by hail prior to harvest. Mom's Bakery has guarded against the risk of the price of wheat rising between April and July.

There is no investment required to enter a futures contract; however, Frank and Mom's Bakery must each post margin with the broker who arranged the contract. The total value is $40,000, so for example, 10% margin would require that they each post a $4000 margin with the broker.

Let us assume that on April 16, the futures price on July 15 delivery of wheat is $3.99.
Then Frank has made money on his contract, since he would in essence get $4.00 per bushel for his wheat, instead of the $3.99 he would expect to get in the absence of his futures contract.

The contract is marked to market daily.[123] Thus Frank's margin account will be increased by: ($.01)(10,000) = $100, while Mom's margin account will be decreased by $100.

Each of them earns one day of interest. Assuming r = 5%, they earn $4000(e^{.05/365} - 1) = \$0.55$.

[118] See Section 5.4 of Derivative Markets by McDonald, on the syllabus of a previous exam.
[119] One can think of mark to market as follows, as the for ward price moves up or down the owner of the futures contract either makes or loses money right away.
In contrast, for a forward contract the profit or loss is realized at expiration.
[120] Futures contracts can be on other assets like gold, stock indices such as the S&P 500, currency such as yen, etc.
[121] *A contract would be for 5000 bushels, so Frank has actually sold two contracts.*
[122] Some futures contracts are settled by delivery, but most are closed out prior to the delivery date.
When you enter into a futures contract you are obligated to make payments if the forward price moves in the wrong direction for you. Thus the effect of the movement in the forward price is captured monetarily on an ongoing basis.
[123] Therefore, Frank will take his profit now, and the new futures price is $3.99.

Thus Frank's margin account is now: \$4000 + \$100 + \$0.55 = \$4100.55.
Mom's margin account is now: \$4000 - \$100 + \$0.55 = \$3900.55.

Exercise: On April 17 the futures price on July 15 delivery of wheat is \$4.02.
What happens to Frank's and Mom's margin accounts?
[Solution: The one day move in price is an increase of \$.03.
Frank's margin account has (\$.03)(10,000) = \$300 subtracted.
He earns one day of interest of: \$4100.55($e^{.05/365}$ - 1) = \$0.56.
Frank's margin account is now: \$4100.55 - \$300 + \$0.56 = \$3801.11.
Similarly, Mom's margin account is now: \$3900.55 + \$300 + \$0.54 = \$4201.09.]

Options on Futures Contracts:

One can buy or sell calls and puts on futures contracts.

If a call is exercised, then the owner of the call acquires a long position in a futures contract with futures price equal to the strike price of the call. The owner of the call if and when he exercises it, will enter into an agreement to accept delivery of the commodity such as corn.
If a call is exercised, then the writer (seller) of the call acquires a short position in a futures contract with futures price equal to the strike price of the call. The writer of the call if and when it is exercised, will enter into an agreement to deliver the commodity such as gold.

If a put is exercised, then the owner of the put acquires a short position in a futures contract with futures price equal to the strike price of the put. The owner of the put if and when he exercises it, will enter into an agreement to deliver the commodity such as oil.
If a put is exercised, then the writer (seller) of the put acquires a long position in a futures contract with futures price equal to the strike price of the put. The writer of the put if and when it is exercised, will enter into an agreement to accept delivery of the commodity such as cattle.

An Example of Using Options on Futures:

We have seen how Frank can use a futures contract to guard against the risk of the price of wheat falling between April and July. Frank could instead buy an option on such a futures contract.[124]

For example, on March 15 Frank could purchase a 3 month put to sell a futures contract on 10,000 bushels of wheat for July delivery. This put would give Frank the option to sell in June such a futures contract at a given strike price.[125]

[124] By buying an option on a futures contract Frank would get price protection without limiting his profit potential.
[125] There are two important dates: when the option expires, May, and the delivery date in the futures contract, July. One could instead of a European option have an American option, to be discussed subsequently. In that case, one could exercise the option at any date up to expiration. *Options on futures contracts are often American, with expiration date one month before the delivery date of the commodity.*

Assume for example the strike price were $4.00 per bushel. Then in June, Frank would have the option to exercise his put and enter into an agreement to deliver wheat in July at $4.00 per bushel. If the futures price in June were more than $4.00 per bushel, then Frank would not exercise his put. If instead the futures price in June were less than $4.00 per bushel, then Frank would exercise his put.

For example, if the futures price in June were $3.80 per bushel, then Frank would exercise his put.[126] He would now have locked in a price of $4.00 per bushel for his wheat, which is better than the $3.80 he could get by entering into a futures contract in June.[127]

The payoff to Frank from the put would be: $4.00 - $3.80 = $0.20 per bushel. The payoff on his put is: (Strike Price - Futures Price)$_+$. This is the same payoff formula as for a put on stock, with the futures price taking the place of the stock price. Thus even though there is no money required to invest in a futures contract, one can apply the same mathematics to options on futures contracts as to options on stocks, with appropriate modifications.[128]
Such a put would protect Frank against the price of wheat declining between March and June. If such a price decline occurs, Frank's wheat is worth less, but his put is worth more.

Similarly, on March 15, Mom's Bakery could purchase a 3 month $4 per bushel strike call to buy a futures contract on 10,000 bushels of wheat for July delivery. This call would give Mom's Bakery the option to buy in June such a futures contract. In June, Mom's would have the option to exercise its call and enter into an agreement to accept delivery of wheat in July at $4.00 per bushel.
If the futures price in June were less than $4.00 per bushel, then Mom's Bakery would not exercise this call. If the futures price in June were more than $4.00 per bushel, then Mom's Bakery would exercise this call.

If for example the futures price in June were $4.10 per bushel, then Mom's would exercise its call and now have entered into a futures contract to receive wheat in July for $4.00 a bushel. Mom's would now have locked in a price of $4.00 per bushel for buying wheat, which is better than the $4.10 it could get via a futures contract. The payoff to Mom's from the call would be: 4.10 - 4.00 = $0.10 per bushel. The payoff on this call is: (Futures Price - Strike Price)$_+$. This is the same payoff formula as for a call on stock, with the futures price taking the place of the stock price.
Such a call would protect Mom's Bakery against the price of wheat increasing between March and June. If such a price increase occurs, Mom's Bakery would have to pay more for wheat, but the call is worth more.

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[126] Alternately, Frank could make money by selling his put which would have increased in value.
[127] In June, Frank could enter into a futures contract for July delivery of wheat with a $3.80 per bushel price. I Instead using his put option on a futures contract, Frank can enter into a futures contract in June for July delivery of wheat with a $4.00 per bushel price.
[128] A form of put-call parity for options on future contracts will be discussed subsequently. In subsequent sections, will be discussed how to price options on futures contracts, using either Binomial Trees or the Black-Scholes formula, in a manner parallel to pricing options on stocks.

Advantages of Using Options on Futures rather than Options on the Underlying Asset:[129]

Options on futures contracts are preferable to options on the underlying asset when it is cheaper and/or more convenient to deliver/receive the futures contract on the asset rather than the asset itself.

It is easier to deliver or receive a futures contract on cattle than it is to deliver or receive the cattle. The futures contract will usually be closed out prior to delivery, and therefore, options on futures contracts are usually settled in cash.

In many situations options on futures contracts tend to have lower transaction costs than options on the underlying asset.

Trading on futures and options on futures are usually conducted side by side on the same exchange, which makes it easier to hedge, etc., and makes the markets more efficient.

Put-Call Parity for Options on Futures Contracts:

$PV[F] = F_{0,T}\, e^{-rT}$.

Therefore, the put-call parity relationship for options on futures contracts is:

$\mathbf{C = P + F_{0,T}\, e^{-rT} - Ke^{-rT}}$.

The put-call parity relationship for options on futures contracts can be obtained from that for options on stocks by: $S_0 \Leftrightarrow F_{0,T}$, and $\delta \Leftrightarrow r$.

Exercise: On March 15 a 3 month \$4 per bushel strike put on a futures contract on 10,000 bushels of wheat for July 15 delivery has a premium of \$815. (The buyer of this put would on June 15 have the option to sell, to the person who wrote the put, a futures contract on 10,000 bushels of wheat for July 15 delivery at \$4 per bushel.)
Currently, the futures price for July 15 delivery of wheat is \$4.20 per bushel.
If r = 5%, what is the premium for the corresponding call?
[Solution: $C = P + F_{0,T}\, e^{-rT} - Ke^{-rT} =$

$\$815 + (10{,}000)(\$4.20)\, e^{-(.05)(1/4)} - (10{,}000)(\$4.00)e^{-(.05)(1/4)} = \$2790$.]

[129] See Options, Futures, and Other Derivatives by Hull, not on the syllabus.

Problems:

8.1 (2 points) A futures contract provides for the delivery of 1000 barrels of Light Sweet Crude Oil in September.
In January, a 4 month $100 per barrel strike call on this futures contract has a premium of $3400.
In January, the futures price for delivery of oil in September is $95 per barrel.
If r = 6%, determine the premium for the corresponding put.
A. $7900 B. $8000 C. $8100 D. $8200 E. $8300

8.2 (2 points) The premium for a 6 month 1500 strike put on a futures contract is 53.
The current futures price is 1600.
If r = 4%, determine the premium for the corresponding call.
A. 130 B. 140 C. 150 D. 160 E. 170

8.3 (3 points) On January 1, Saul enters into a futures contract on 5000 bushels of soybeans at $12.40 per bushel for delivery in August. Saul has a long position; he has agreed to accept delivery of soybeans in August. Saul post a margin of 10%. The interest rate is 6%.
The futures prices for August delivery are on the following dates:

January 1	$12.40
January 2	$12.60
January 3	$12.55
January 4	$12.30
January 5	$12.40

Determine the amount in Saul's margin account on each of these days.

8.4 (1 point) The premium for an at-the money call on a futures contract is $5. What is the premium for an at-money put on the same futures contract with the same time until maturity?

Section 9, Synthetic Positions[130]

Put-call parity followed from creating a synthetic position which matched the cashflows of another investment. We made use of the **"law of one price" which states that two positions that generate the exact same cashflows should have the same cost**. Creating synthetic positions can be useful to derive results, help ones understanding, and may also have practical applications in investing.

Synthetic Share of Stock:

We can create a synthetic share of stock by buying a call, selling a put, lending the present value of the strike price K, and lending the present value of any dividends.

Exercise: Show that this position is equivalent to buying the stock now and holding it until time T.
[Solution: If we buy and hold the stock, then we will receive the dividend payments and own the stock at time T. In the above position, we will receive the dividend payments as that loan is repaid. Also we will be paid back K at time T. If $S_T \geq K$, then we use our call to buy a share for K.
If instead $S_T < K$, then we buy a share of Stock for K from the person to whom we sold the put. In either case, we own a share of stock at time T, as well as having received the dividend payments.]

This shows that $S_0 = C_{Eur}(K, T) - P_{Eur}(K, T) + K e^{-rT} + PV[Div]$, one of the put-call parity relationships discussed previously.

One way to remember how to create a given synthetic position is to write the put-call parity relationship with the desired item alone on the righthand side of the equation.

If the stocks pays dividends continuously, then $S_0 = e^{\delta T} \{C_{Eur}(K, T) - P_{Eur}(K, T) + K e^{-rT}\}$.

We can create a synthetic share of stock by: buying $e^{\delta T}$ calls, selling $e^{\delta T}$ puts, lending $K e^{-(r-\delta)T}$.

[130] See page 285 of Derivative Markets by McDonald.

<u>Synthetic Treasury Bill</u>:

$K e^{-rT} + PV[Div] = S_0 - C_{Eur}(K, T) + P_{Eur}(K, T)$.

Therefore, we can create a synthetic T-Bill by buying the stock, selling a call, and buying the put. This is called a conversion. One is lending money.

If $S_T > K$, then we sell our share for K to the person who bought the call. If instead $S_T \le K$, then we sell a share of stock for K using our put. In either case, we have K at time T.
Thus for an investment now, we get K at time T. This is equivalent to a risk free Treasury Bill that returns K at time T, plus we get the present value of any dividends on the stock.

Alternately, one could also borrow the present value of dividends.
This is equivalent to a risk free Treasury Bill that returns K at time T, since:
$Ke^{-rT} = S_0 - C_{Eur}(K, T) + P_{Eur}(K, T) - PV[Div]$.

Alternately, if dividends are paid continuously, then one could instead buy exp[-δT] shares of stock, and reinvest the dividends. Then we would end up with one share of stock at time T. This is equivalent to a risk free Treasury Bill that returns K at time T, since
$Ke^{-rT} = S_0e^{-\delta T} - C_{Eur}(K, T) + P_{Eur}(K, T)$.

One can create a synthetic <u>short</u> T-Bill position by shorting the stock, buying a call, and selling the put. This is called a reverse conversion. One is borrowing money.

<u>Synthetic Options</u>:

We can create a synthetic call by: buying the stock, buying a put, borrowing the present value of the strike price, and borrowing the present value of any dividends.

Exercise: Show that this position is equivalent to buying the call.
[Solution: We repay one loan as we receive the dividend payments.
If $S_T \ge K$, then we sell our share for S_T, repay the loan by paying K, and are left with $S_T - K$.
If instead $S_T < K$, then we use our put to sell our share for K, repay the loan by paying K, and are left with 0. These are the same payoffs as if we had purchased the call.]

This shows that $C_{Eur}(K, T) = S_0 + P_{Eur}(K, T) - K e^{-rT} - PV[Div]$, one of the put-call parity relationships discussed previously.

We can create a synthetic put by buying a call, shorting the stock, lending the present value of the strike price, and lending the present value of any dividends.

Exercise: Show that this position is equivalent to buying the put.
[Solution: We borrow a share of stock, sell it for S_0, and will give this person a share of stock when the option expires. We also must pay this person the stock dividends they would have gotten on the stock, when they would have gotten them. We pay the dividend payments from one of the loans. If $S_T \geq K$, then we buy a share for K, using our call and the money from the other loan. After returning the share to the person we borrowed it from, we are left with nothing.
If instead $S_T < K$, then we buy a share for S_T. After returning the share to person we borrowed it from, we are left with $K - S_T$. These are the same payoffs as if we had purchased the put.]

This shows that $P_{Eur}(K, T) = C_{Eur}(K, T) - S_0 + K e^{-rT} + PV[Div]$, one of the put-call parity relationships discussed previously.

In the case of continuous dividends, $P_{Eur}(K, T) = C_{Eur}(K, T) - e^{-\delta T}S_0 + K e^{-rT}$.

Thus we can create a synthetic put by buying a call, shorting $e^{-\delta T}$ shares of stock, and lending the present value of the strike price.

$C_{Eur}(K, T) = P_{Eur}(K, T) + e^{-\delta T}S_0 - K e^{-rT}$.

Thus we can create a synthetic call by buying a put, buying $e^{-\delta T}$ shares of stock, and borrowing the present value of the strike price.

Synthetic Forward Contract:[131]

We can create a synthetic forward contract by buying a call and selling a put.

If $S_T \geq K$, then we buy a share for K, using our call.
If instead $S_T < K$, then we buy a share for K, from the person to whom we sold the put.
Thus we have a forward contract to buy the stock at price K. This has present value $PV_{0,T}[F_{0,T} - K]$.
Therefore, $PV_{0,T}[F_{0,T} - K] = C_{Eur}(K, T) - P_{Eur}(K, T)$, as discussed previously.

[131] See page 282 of Derivative Markets by McDonald.

Problems:

9.1 (2 points) The price of a stock is $135. The stock pays dividends at the continuous rate of 2%. The price of a European call with a strike price of $140 is $17.39 and the price of a European put with a strike price of $140 is $19.14. Both options expire in eight months.
Calculate the annual continuously compounded risk-free rate on a synthetic T-Bill created using these options.
A. Less than 3%
B. At least 3%, but less than 3%
C. At least 4%, but less than 4%
D. At least 5%, but less than 6%
E. At least 6%

9.2 (2 points) Consider options on a non-dividend paying stock.
The price of a 3 year European call with a strike price of $100 is $39.70.
The price of a 3 year European put with a strike price of $100 is $8.23.
r = 6%.
Calculate the price of a synthetic share of stock created using these options.
A. 100 B. 105 C. 110 D. 115 E. 120

9.3 (3 points) The stock of Rich Resorts has a current price of 150, and pays no dividends.
The stock of Cereal Growers has a current price of 80, and pays no dividends.
This coming summer will either be dry or wet.
If the summer is dry, then 4 months from now Rich Resorts stock price will be 180, and Cereal Growers stock price will be 70.
If the summer is wet, then 4 months from now Rich Resorts stock price will be 130, and Cereal Growers stock price will be 90.
What is the continuously compounded risk free rate?
A. 3.25% B. 3.50% C. 3.75% D. 4.00% E. 4.25%

9.4 (2 points) Consider options on a dividend paying stock.
The price of a 9-month European call with a strike price of $60 is $5.67.
The price of a 9-month European put with a strike price of $60 is $7.52.
r = 7%.
Using these options you create a synthetic 9-month forward contract on a share of stock.
Calculate the price, with payment on delivery, of this forward contract.
A. 55 B. 56 C. 57 D. 58 E. 59

9.5 (CAS3, 5/07, Q.13) (2.5 points) The price of a non-dividend paying stock is $85.
The price of a European call with a strike price of $80 is $6.70 and the price of a European put with a strike price of $80 is $1.60.
Both options expire in three months.
Calculate the annual continuously compounded risk-free rate on a synthetic T-Bill created using these options.

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%

Section 10, American Options

An American style option may be exercised at any time up to the expiration date.

For example, Jim buys a 3 year American put option on ABC Stock with a strike price of $100. This gives Jim the right to sell a share of ABC Stock for $100 at any time during the next 3 years.[132]

American Versus European Options:[133]

Since Jim may choose to exercise his American option prior to its expiration, an American option may be worth more than a European option. Since Jim may choose to never exercise early and instead only exercise his American option on its expiration date, **an American option is worth at least as much as the similar European option.**

The put-call parity relationships discussed previously for the European options do not hold for an American option.[134]

Maximum and Minimum Prices:

As discussed previously, $S_0 \geq C_{Eur} \geq (PV_{0,T}[F_{0,T}] - PV_{0,T}[K])_+ \geq 0$.

In fact, since a European call can only be exercised at expiration: $PV_{0,T}[F_{0,T}] \geq C_{Eur}$.

Using the same reasoning as used previously with respect to a European option, an American call option can not be worth more than the current stock price. Therefore, since $C_{Amer} \geq C_{Eur}$ we have:

$$\mathbf{S_0 \geq C_{Amer}(S_0, K, T) \geq C_{Eur}(S_0, K, T) \geq (PV_{0,T}[F_{0,T}] - PV_{0,T}[K])_+}.$$ [135]

In fact, since an American call can be exercised right away: $C_{Amer} \geq (S_0 - K)_+$.[136]

[132] Recall that for a European style option, Jim would get the option to sell a share of stock on only a single day.

[133] See Section 9.3 of Derivatives Markets by McDonald.

[134] Appendix 9.A of McDonald, not on the syllabus, discusses parity bounds for American options.
$C_{Amer}(S_0, K, T) + K - PV_{0,T}[F_{0,T}] \geq P_{Amer}(S_0, K, T) \geq C_{Amer}(S_0, K, T) + PV_{0,T}[K] - S$.
$P_{Amer}(S_0, K, T) + S - PV_{0,T}[K] \geq C_{Amer}(S_0, K, T) \geq P_{Amer}(S_0, K, T) + PV_{0,T}[F_{0,T}] - K$.
Note that for European Options, $C_{Eur}(S_0, K, T) + K - PV_{0,T}[F_{0,T}] \geq C_{Eur}(S_0, K, T) + PV_{0,T}[K] - PV_{0,T}[F_{0,T}]$ $= P_{Eur}(S_0, K, T) \geq C_{Eur}(S_0, K, T) + PV_{0,T}[K] - S$.

[135] See Equation 9.9 in McDonald.

[136] Not discussed in Derivatives Markets by McDonald. If $S_0 > K$, in other words if the call is in the money, then one could exercise the American call immediately, getting a payoff of $S_0 - K$.
If the dividend rate is small, then $PV_{0,T}[F_{0,T}] - PV_{0,T}[K] > S_0 - K$.
If the “dividend rate” were large, such as when dealing with options on currency, the reverse could be true.

As discussed previously, $K \geq P_{Eur} \geq (PV_{0,T}[K] - PV_{0,T}[F_{0,T}])_+ \geq 0$.

In fact, since a European put can only be exercised at expiration: $PV_{0,T}[K] \geq P_{Eur}$.[137]

Using the same reasoning as used previously with respect to a European option, an American put option can not be worth more than the strike price. Therefore, since $P_{Amer} \geq P_{Eur}$ we have:

$$\mathbf{K \geq P_{Amer}(S_0, K, T) \geq P_{Eur}(S_0, K, T) \geq (PV_{0,T}[K] - PV_{0,T}[F_{0,T}])_+}.$$ [138]

In fact, since an American put can be exercised right away: $P_{Amer} \geq (K - S_0)_+$.[139]

Time Until Expiration:

Owen buys an American option with one year to expiration.
Tom buys an otherwise similar American option with two years to expiration.
Tom decides to exercise his option whenever Owen exercises his option within the first year, and Tom throws his option away if he still holds it at the end of one year and it is not in the money.[140]

If Tom mirrors Owen actions, then Tom's option is worth the same as Owen's option.
In general, **an American option is worth at least as much as a similar option with less time to expiration**.

If instead Tom employs a better strategy, then his option would be worth more than Owen's. Tom's option provides more opportunities for action than Owen's option with a shorter time until expiration. This is not true for a European option.

Olivia buys a one year European option and Tracy buys an otherwise similar two year European option. Olivia can only exercise her option at time = 1, while Tracy can only exercise her option at time = 2. While Tracy's option is usually worth more, this is not always the case.

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137 See MFE Sample Exam Q. 26.
138 See Equation 9.10 in McDonald.
139 Not discussed in Derivatives Markets by McDonald. See MFE Sample Exam Q. 26.
If $S_0 < K$, in other words if the put is in the money, then one could exercise the American put immediately, getting a payoff of $K - S_0$.
140 Mirroring Owen is not an optimal strategy for Tom.

Exercise: Olivia and Tracy have purchased puts with strike prices of $100.
At time = 1 the stock is worth $0.01. Whose option is worth more?
[Solution: Olivia exercises her option at time = 1 and makes $99.99.
The present value is $99.99e^{-r}.
The most Tracy can make by exercising her option at time = 2 is $100.
The present value is $100e^{-2r}, less than $99.99e^{-r} for any reasonable value of r.
Comment: We have shown that it is possible that a European put could be worth less than a similar option with less time until expiration.]

Exercise: Olivia and Tracy have purchased calls with strike prices of $100.
At time = 1 the stock is worth $125. At time equal 1.1 the stock will pay a dividend of $120.
Whose option is worth more?
[Solution: Olivia exercises her option at time = 1 and makes $25.
After paying the very large dividend, the price of the stock will decline to something in the range of $5. It is extremely unlikely that the price of the stock by time = 2 will exceed 100. At time = 2, the only time Tracy can exercise her option, it is very unlikely that her call option is in the money.
Tracy's option will almost always turn out to be worthless.
Comment: We have shown that it is possible when dividends are paid that a European call could be worth less than a similar option with less time until expiration.]

For European calls on stocks that do not pay dividends, the premium is the same the corresponding American call. Therefore, **for a European call on a given non-dividend paying stock, the call is worth at least as much as a similar call with less time to expiration.**

However, the same relationship does not necessarily hold for either European calls on stocks that pay dividends, or for European puts.[141]

Early Exercise:

It turns out that for an American option, depending on the current price of the stock, the strike price, and the time until expiration, etc., sometimes it is worthwhile to exercise the option prior to the expiration date, and sometimes it is not. Here we are talking about the present value of each strategy, rather than a 20-20 hindsight view of what actually turned out to happen to the stock price after one had made a decision.

As will be shown subsequently, **if the stock pays no dividends, then it is never worthwhile to exercise an American call option early.**[142]

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[141] See page 297 of Derivative Markets by McDonald.
[142] If the stock does pay dividends, then we are only concerned about any dividends paid between the time the option is bought and the time it expires.

Thus for a stock that pays no dividends, an American call option is worth the same as a European call option.

As discussed previously, an American option is worth at least as much as a similar American option with less time to expiration.
Therefore, for a stock that pays no dividends, a European option is worth at least as much as a similar option with less time to expiration.

Difficulty of Deciding Whether to Exercise Early:

There are various simple strategies one could use to decide when to exercise an American Option. For example, one could choose to always exercise the option right away. Jim buys a 3 year American put option on ABC Stock with a strike price of $100. If the current market price of ABC Stock is $120, this is a stupid strategy, since Jim would be better off selling the stock for the $120 market price rather than the $100 strike price.

Assume that in one year, it turns out that ABC Stock has a market price of $90. If Jim buys a share of ABC Stock for $90 and then exercises his option to sell at $100, he makes $10. However, this may or may not be the best decision. If over the next two years, the price of ABC Stock never falls below $90, then Jim should have exercised his option early. If over the next two years, the price of ABC Stock falls to $80, then Jim could have made $20 if he waited to exercised his option.

Out of the many possible paths, let us look at four, all of which have a price of $90 at time 1:[143]

Time	Price Path 1	Price Path 2	Price Path 3	Price Path 4
1	$90	$90	$90	$90
2	$95	$90	$85	$85
3	$100	$90	$80	$100

In path #1, Jim should exercise his option at time 1. If he instead waits until time 2 to exercise his option, then he makes only $5 rather than $10, and also loses the time value of money. If he does not exercise his option until expiration, he is out of luck and makes nothing!

In path #2, Jim should exercise his option at time 1. If he waits to exercise his option, he still makes $10, but loses the time value of money.

In path #3, Jim should wait to exercise his option. If he waits until time 2 he makes $15 instead of $10. If he waits until time 3 he makes $20!

In path #4, Jim should also wait to exercise his option. If he waits until time 2 he makes $15 instead of $10. However, if he then waits until time 3 he makes nothing!

[143] Jim can exercise his option on any day through expiration. I have only looked at three points in time for simplicity.

Assuming Jim can not see the future, it can be a tough decision whether or not to exercise his American option early. Ideally, Jim should exercise his option whenever the value of doing so is greater than the present value of not doing so.

Thus a key tool would be some way for Jim to estimate the present value of his put option based on the current stock price, time to expiration, strike price, etc. In subsequent sections, we will discuss ways to tackle this problem.

Early Exercise of a Call:[144]

The continuation value of an option is the value of the option at a given point in time and at a given stock price, if we do not exercise the option immediately.
One should exercise early if the continuation value of the option is less than the value of exercising the option. In other words, one should exercise a call at time $t < T$ if $C_{Amer}(S_t, K, T-t) < S_t - K$.

If one exercises an American call before expiration, one will own the stock, and benefit from dividends. If there is a particularly large stock dividend, this would make it worthwhile to exercise the call just before the payment of the dividend.[145]

By exercising an American call before expiration, one loses interest one can earn on the strike price. Also by exercising early, one loses the "implicit protection against the stock price moving below the strike price." If one exercises the call early and buys the stock, we take on the risk that the stock price might go down a lot. We can understand this by examining the value of a similar European option.

Put-call parity states that for a stock that pays dividends continuously, at time t:

$C_{Amer}(S_t, K, T - t) \geq C_{Eur}(S_t, K, T - t) = P_{Eur}(S_t, K, T - t) + S_t e^{-\delta(T-t)} - K\, e^{-r(T-t)}$

$= (S_t - K) + P_{Eur}(S_t, K, T - t) + K(1 - e^{-r(T-t)}) - S_t\{1 - e^{-\delta(T-t)}\}$

= (Exercise Value of the Call) + (Insurance against $S_T < K$) + (Time value of money on K)
- (Present Value of future Dividends).

For a stock that does not pay dividends, put-call parity states that at time t:

$C_{Amer}(S_t, K, T - t) = C_{Eur}(S_t, K, T - t) = (S_t - K) + P_{Eur}(S_t, K, T - t) + K\{1 - e^{-r(T-t)}\}$

= (Exercise Value of the call) + (Insurance against $S_T < K$) + (Time value of money on K).[146]

[144] See pages 294 to 296 and Section 11.1 of Derivatives Markets by McDonald.
[145] A dividend is paid to the person who owns the stock on a specific day. If for example that day is March 30, and if one buys the stock on March 31, then the price is ex-dividend. If Dave buys the stock from Judy on March 31, Judy will receive the dividend, even if the dividend is actually paid in early April.
[146] See equation 9.11 in Derivatives Markets by McDonald.

$P_{Eur}(S_t, K, T - t) \ge 0$ and $K(1 - e^{-r(T-t)}) \ge 0$.

Therefore, for a stock that does not pay dividends:
$C_{Amer}(S_t, K, T - t) \ge S_t - K$.
Therefore, there is never a reason to exercise an American call early in the absence of dividends.

For American calls there are two reasons to wait and one reason to exercise early:
1. The time value of the strike price is lost if one exercises early.
2. The implicit insurance protection against the stock price moving below the strike price is lost if one exercises early.
3. The value of dividends is gained if one exercises early.

If we exercise early, then we spend the strike price. If instead we were to wait to exercise, then we do not spend the strike price and we can earn interest on that money.
If we exercise early, then we own the stock, and risk the stock price going down. If instead we were to wait to exercise, then we retain this implicit insurance protection against the stock price moving below the strike price.[147] Put another way, if we wait to exercise the call, we avoid the possibility of capital losses on the stock.
If we exercise the call early, then we own the stock and get the dividends.

$C_{Amer}(S_t, K, T - t) \ge C_{Eur}(S_t, K, T - t) = P_{Eur}(S_t, K, T - t) + PV_{t,T}[F_{t,T}] - PV_{t,T}[K]$
$\ge PV_{t,T}[F_{t,T}] - PV_{t,T}[K]$. Therefore, $C_{Amer} \ge PV[F_{t,T}] - PV[K]$, as discussed previously.

The payoff for exercising early an American call at time t is:
$S_t - K = PV[F_{t,T}] + PV[Div] - PV[K] - (K - PV[K])$
$= PV[F_{t,T}] - PV[K] + PV[Div] - (K - PV[K]) \le C_{Amer}(S_t, K, T - t) + PV[Div] - (K - PV[K])$.

Therefore, if $PV_{t,T}[Div] - (K - PV_{t,T}[K]) < 0$, then $S_t - K \le C_{Amer}$,
and it would not be worthwhile to exercise early.

If $K - PV_{t,T}[K] > PV_{t,T}[Div]$, then one should <u>not</u> exercise an American call early at time t.[148] [149]

For example, if K = \$100, r = 5%, and there are 6 months to expiration of an American call, then if $PV[Div] < 100(1 - e^{-.025}) = \$2.47$, it would not be worthwhile to exercise at this time. For example, if there is a single \$2 dividend tomorrow it would not be worthwhile to exercise at this time.

[147] Personally, I do not find McDonald's use of the term "insurance implicit in the call" to be helpful.
[148] See Equation 9.12 in Derivative Markets by McDonald. $K - PV_{t,T}[K] = K(1 - \exp[-r(T- t)])$
Specifically, if there are no dividends, then one should not exercise an American call early.
[149] If K - PV[K] < PV[Div], then it may or may not make sense to exercise an American call early.
Another necessary condition to exercise early an American call is that $S_t > K$.

If instead, PV[Div] > $2.47, then it might be worthwhile to exercise at this time. For example, if there is a single $3 dividend two months from now, then PV[Div] = $3e^{-.05/6}$ = 2.98 > 2.47, and it might be worthwhile to exercise at this time.

In fact, $C_{Amer}(S_t, K, T - t) \geq P_{Eur}(S_t, K, T - t) + PV_{t,T}[F_{t,T}] - PV_{t,T}[K]$.
Therefore, the payoff for exercising early an American call at time t is:
$S_t - K = PV[F_{t,T}] + PV[Div] - PV[K] - (K - PV[K]) = PV[F_{t,T}] - PV[K] + PV[Div] - (K - PV[K])$
$\leq C_{Amer}(S_t, K, T - t) - P_{Eur}(S_t, K, T - t) + PV[Div] - (K - PV[K])$.

Therefore, if $PV_{t,T}[Div] - (K - PV_{t,T}[K]) - P_{Eur}(S_t, K, T - t) < 0$, *then* $S_t - K \leq C_{Amer}$,
and it would not be worthwhile to exercise early.
If $P_{Eur}(S_t, K, T - t) + K - PV_{t,T}[K] > PV_{t,T}[Div]$, *then one should not exercise an American call early at time t.*[150] *If were to exercise an American Call early, then we would own the stock and collect its dividends, but we would lose the interest on the strike price and also lose the implicit insurance protection represented by the put premium.*

Let us assume there is a single $3 dividend two months from now.
Jubal and Anderson each have identical 6 month $100 strike American calls.
Jubal exercises his call now, pays $100 and owns the stock.
Anderson instead invests $100 $e^{-.05/6}$ = $99.17 at the risk free rate for two months.

Two months from now, just before the dividend is paid, Anderson has $100 and uses it exercises his call and own the stock.
At that point in time both Jubal and Anderson own the stock and get the dividend payment.
However, Jubal invested $100 today, while Anderson only invested $99.17.

Thus Anderson's strategy was better.[151]

In general, **if it is optimal to exercise an American call early, and dividends are paid at discrete times, then it is best to exercise right before the payment of a (large) dividend**, in other words at the last moment before the ex-dividend date.[152]

[150] Not discussed in Derivative Markets by McDonald.
[151] While Anderson does better than Jubal, it may still not be optimal to exercise the call early.
[152] Note that this statement is conditional on it being optimal to exercise the call early.

Early Exercise of a Put:[153]

The situation for early exercise of puts is somewhat reversed from calls.

For American puts there are two reasons to wait and one reason to exercise early:
1. The time value of the strike price is gained if one exercises early.
2. The implicit insurance protection against the stock price moving above the strike price is lost if one exercises early.
3. The value of dividends is lost if one exercises early.

If we exercise early, then we receive the strike price and we can earn interest on that money.
If we exercise the put early and use it to sell the stock, then we do not own the stock, and fail to gain if the stock price goes up. If instead we were to wait to exercise, then we retain this implicit insurance protection against the stock price moving above the strike price. Saying it another way, if we wait to exercise the put, we retain the possibility of additional capital gains on the stock.
If we exercise the put early, then we do not own the stock and do not get the dividends.

It can make sense to exercise an American put early, whether or not the stock pays dividends.

By Put-Call Parity: $P_{Eur} = Ke^{-rT} - S_T + PV[Div] + C_{Eur}$.

$P_{Amer} \geq P_{Eur}$. $\Rightarrow$

$P_{Amer} \geq K - S_T + PV[Div] + C_{Eur} - K(1 - e^{-rT})$ =

(Exercise Value of the Put) + (Present Value of Future Dividends)
+ (Insurance against $S_T > K$) - (Time Value of Money on Strike).

Moneyness:[154]

If at a point in time it is worthwhile to exercise an American call with a strike price of K, it also makes sense to exercise an otherwise similar call with a strike price lower than K. For example, if one should exercise an option to buy at 100, then one should also exercise a similar option to buy at 80.

If at a point in time it is worthwhile to exercise an American put with a strike price of K, it also makes sense to exercise an otherwise similar put with a strike price higher than K. For example, if one should exercise an option to sell at 60, then one should also exercise a similar option to sell at 70.

If it makes sense to exercise an option that is in the money, then it also makes sense to exercise an option that is more in the money.

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[153] See pages 296 to 297 and Section 11.1 of Derivatives Markets by McDonald.
[154] See page 304 of Derivatives Markets by McDonald.

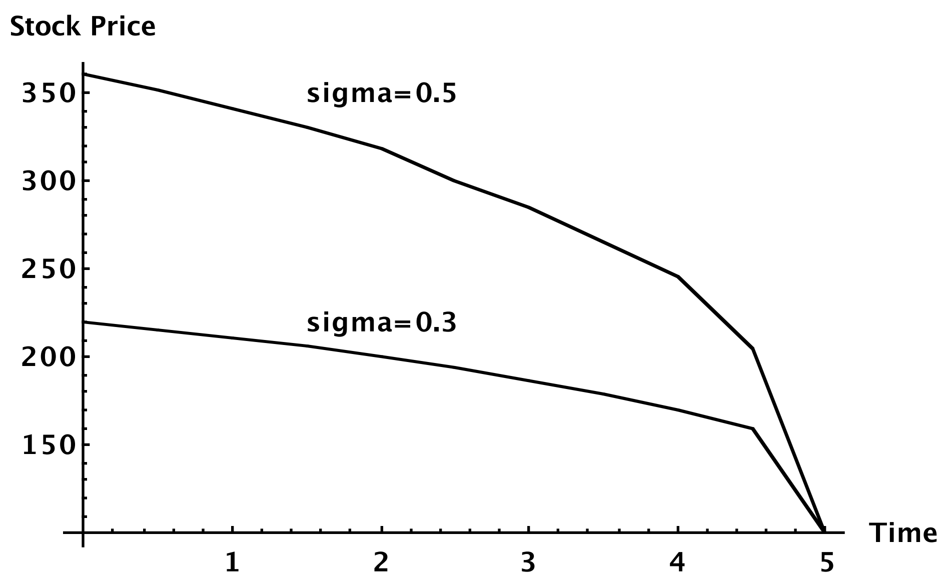
Early Exercise Boundaries:

For an American option, using for example Binomial Trees to be discussed subsequently, one can determine the stock price at which one should exercise early.

In the case of a call, one would exercise early if the stock price were greater than a given value. For example, at time 1.5 one might exercise a call early if the stock price were more than 200. In the case of a put, one would exercise early if the stock price were less than a given value. For example, at time 3 one might exercise a put early if the stock price were less than 55.

For a given option, as a function time, the set of stock prices at which the continuation value is equal to the value of immediate exercise is called the early exercise boundary.

The following graph shows early exercise boundaries for a 5-year 100 strike American call, for σ either 30% or 50%:[155] [156]



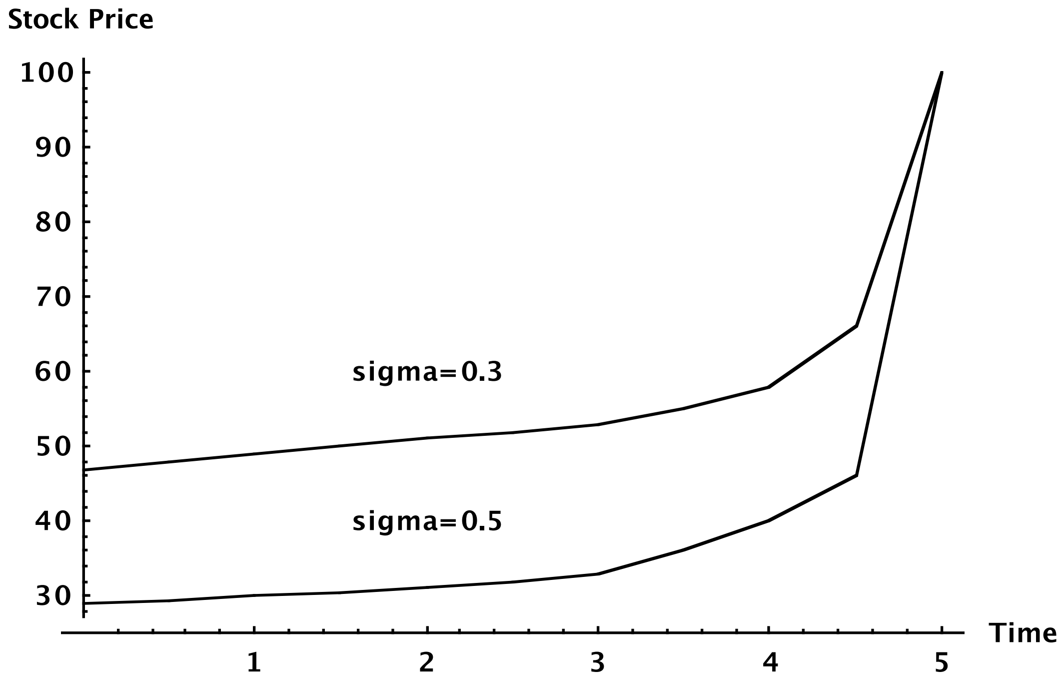
We note two features. First, as σ increases, the exercise boundary is higher. For larger σ, the insurance protection is larger. Therefore, the larger σ, the larger the continuation value. Therefore, a higher stock price, and thus a higher value of exercising the call immediately, is required in order to make it worthwhile to exercise the option early.

Second, as the option gets closer to expiration, the exercise boundary approaches the strike price. As the option approaches expiration, we lose less insurance protection by exercising early.

[155] Based on Figure 11.1 of Derivatives Markets by McDonald, which was computed using Binomial Trees, to be discussed subsequently.

[156] While one needs to know how to work with Binomial Trees, the huge amount of work required to determine the exercise boundary is well beyond what you could be required to do on the exam, even if one were supplied with a computer!

The following graph shows early exercise boundaries for a 5-year 100 strike American put, for σ either 30% or 50%:[157]



We note two features. First, as σ increases, the exercise boundary is lower. For larger σ, the insurance protection is larger. Therefore, the larger σ, the larger the continuation value. Therefore, a lower stock price, and thus a higher value of exercising the put immediately, is required in order to make it worthwhile to exercise the option early.

Second, as the option gets closer to expiration, the exercise boundary approaches the strike price. As the option approaches expiration, we lose less insurance protection by exercising early.

For both puts and calls, the early-exercise criterion becomes less stringent as the option has less time until expiration.
For both puts and calls, the early-exercise criterion becomes more stringent as the volatility of the stock increases.

Put-Call Parity:[158]

The put-call parity relationships discussed previously for the European options do not hold for an American option. However, it is the case that for American Options:

S_0 - PV[Div] - K ≤ C_{Amer} - P_{Amer} ≤ S_0 - Ke^{-rT}.

[157] Based on Figure 11.2 of Derivatives Markets by McDonald, which was computed using Binomial Trees, to be discussed subsequently.
[158] Appendix 9.A of McDonald, not on the syllabus, discusses parity bounds for American options.

Problems:

10.1 (2 points) r = 0%. Stock ABC pays dividends. You own an American call on ABC.
Is it ever optimal to exercise this option before expiration? Briefly explain why.

10.2 (1 point) You have an American put option to exchange one share of Stock A for one share of Stock B. Neither stock pays dividends. In what circumstances might you exercise this option early?

10.3 (1 point) Which of the following statements about American style options is not true?

A. The option may be exercised at any time during its life.
B. If the underlying stock does not pay a dividend, one should not exercise a call option early.
C. The option is worth at least as much as a similar European style option.
D. The option is worth at least as much as a similar option with more time to expiration.
E. All of A, B, C, and D are true.

10.4 (2 points) On March 1, ABC company stock is worth 20.
However, ABC has announced that on April 1 it will pay a liquidating dividend; ABC will pay its entire value to its stockholders.
On March 1, compare the values of a European 10-strike call on XYZ stock that expires March 15 and a similar option that expires April 15.

10.5 (2 points) r = 0%. Stock ABC pays dividends. You own an American put on ABC.
Is it ever optimal to exercise this option before expiration? Briefly explain why.

10.6 (1 point) An American 85 strike 18 month put is an option on a stock with current price 81 and forward price of 84. r = 3%.
Which of the following intervals represents the range of possible premiums for this option?
A. [0, 81] B. [0, 84] C. [0, 85] D. [0.96, 84] E. [0.96, 85]

10.7 (1 point) Which of the following statements is true?
1. A European put is worth at least as much as a similar put with more time until expiration.
2. A European call is worth at least as much as a similar call with more time until expiration.
3. A European option is worth at most as much as a similar American style option.
A. 1 B. 2 C. 3 D. 1, 2, 3 E. None of A, B, C, or D

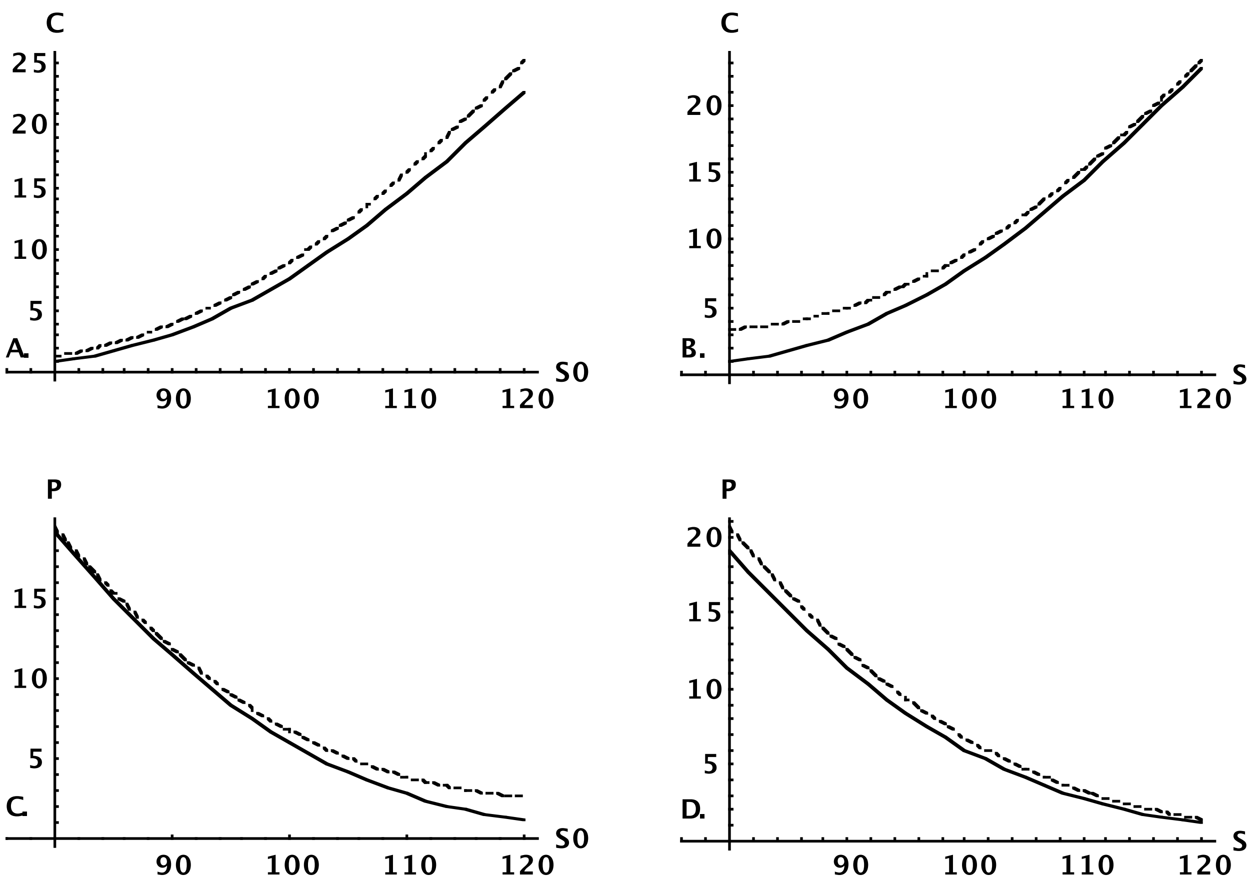
10.8 (1 point) You have an American call option to exchange one share of Stock A for one share of Stock B. Neither stock pays dividends. In what circumstances might you exercise this option early?

10.9 (3 points) On January 1 you purchase a 1 year $90 strike American call. r = 6%.
The stock will pay dividends of $1 each on: March 1, June 1, September 1, and December 1.
At what points in time might it be optimal to exercise your call?

10.10 (2 points) XYZ company goes bankrupt on January 1 and its stock becomes worthless. On January 1, compare the values of a European 100-stirike put on XYZ stock that expires April 1 and a similar option that expires July 1.

<u>10.11</u> (1 point) You have an American call option with strike price of \$100. r = 5% and δ = 2%. If the stock price had zero volatility, under what circumstances should you exercise this option early?

<u>10.12</u> (2 points) r = 6%.
Consider otherwise similar American and European options on a stock that does not pay dividends. Which of the following could be a graph of the option premiums as a function of the initial stock price, holding everything else constant?
In each case, the American option premium is shown as dashed.



E. None of A, B, C, or D

10.13 (2 points) Briefly discuss early exercise of American options if $\sigma = 0$.

10.14 (1 point) An American 95 strike 3 year call is an option on a stock with current price 90 and forward price of 100. r = 5%. Determine the width of the range of possible prices of this option.
A. 84 B. 86 C. 88 D. 90 E. 92

<u>**10.15**</u> (4 points) Use the following information:

- A 2-year American put option on Euros has K = $1.30.
- $r_\$ = 4\%$.
- $r_€ = 6\%$.

Draw and label a diagram to show bounds (ignoring transaction costs) on the possible values of the American put as a function of the exchange rate when the option is bought.
The bounds illustrated should be independent of any model of the movement of exchange rates.
You should explain the rationale behind the bounds shown.

10.16 (2 points) You own a six-month 50-strike American call option on a stock.
The stock pays dividends at the continuously compounded annual rate of 2%.
The stock currently sells at 54, and is expected to sell at either 50 or 62 six months from now.
What annual continuously compounded risk-free interest rate would equate the value of exercising the call immediately with the present value of waiting until the end of the term?
A. 1% B. 2% C. 3% D. 4% E. 5%

<u>**10.17**</u> (2 points) Which of the following statements is true?
i. For an American option, the early exercise criterion becomes less stringent closer to expiration.
ii. For American calls, the early exercise boundary is higher for larger volatility.
iii. For American puts, the early exercise boundary is higher for larger volatility.
(A) Only (i) is true
(B) Only (ii) is true
(C) Only (i) and (ii) are true
(D) Only (i) and (iii) are true
(E) (i), (ii) and (iii) are true

10.18 (1 point) An American 70-strike six-month put is an option on a stock with current price of 74.
$\delta = 1\%$. r = 4%.
Determine the range of possible put premiums.

10.19 (4 points) Use the following information:

• A 5-year American call on a stock has K = 100.

• r = 4%.

• δ = 3%.

Draw and label a diagram to show bounds (ignoring transaction costs) on the possible values of the American call as a function of the stock price when the option is bought.
The bounds illustrated should be independent of any model of the movement of stock prices.
You should explain the rationale behind the bounds shown.

10.20 (5B, 11/98, Q.35) (2 points) You have purchased a two-year American call option on a stock that pays an annual dividend. The dividend will always equal 10% of the share price and the next dividend is payable one year from today. The exercise price is equal to the current share price. Each year the stock may increase in value by 20% or decrease in value by 20%. The value of the stock will drop by the dividend amount immediately after it is paid. The continuously compounded risk-free rate is 5%.
Assuming that investors are indifferent to risk, under what circumstances would you exercise the option prior to expiration? Explain.

10.21 (5B, 5/99, Q.34) (1 point) You own a one-year American put option on a non-dividend paying stock with an exercise price of $42. The stock currently sells at $40, and is expected to sell at either $38 or $48 one year from now. What annual continuously compounded risk-free interest rate would equate the value of exercising the put immediately with the present value of waiting until the end of the term? Show all work.

10.22 (IOA 109 Specimen Exam 5/99, Q.5) (3.75 points) Draw a diagram to show bounds (ignoring transaction costs) on the possible values of an American call option as a function of the price of the underlying security. The bounds illustrated should be independent of any model for the price process followed by the underlying security. You should explain the rationale behind the bounds shown.

10.23 (5B, 11/99, Q.15) (1 point)
For which of the following options might it be rational to exercise before maturity?
1. American put on a dividend-paying stock
2. American put on a non-dividend-paying stock
3. American call on a dividend-paying stock
A. 1 B. 3 C. 1, 2 D. 2, 3 E. 1, 2, 3

10.24 (MFE Sample Exam, Q.26)

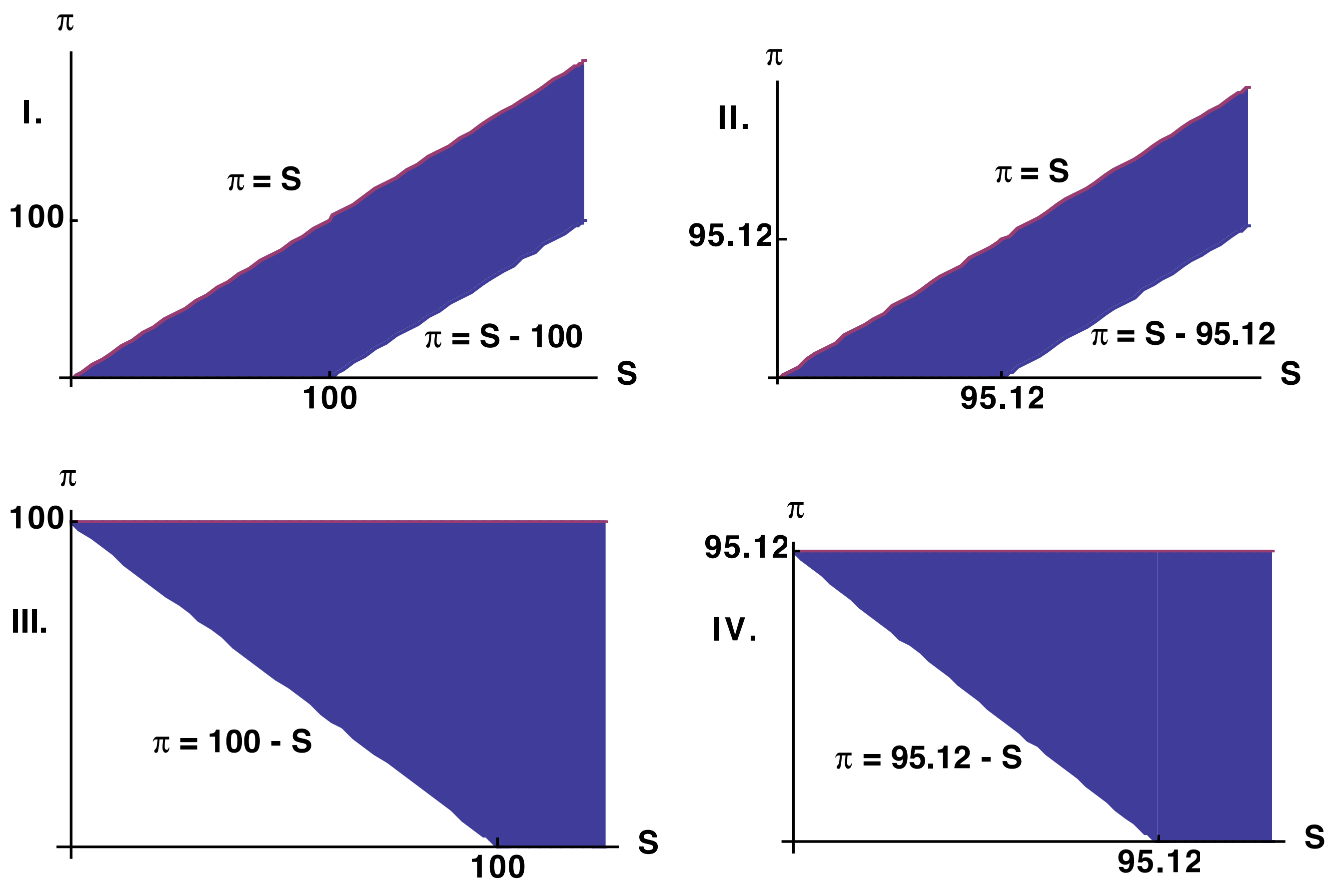
Consider European and American options on a nondividend-paying stock. You are given:

(i) All options have the same strike price of 100.

(ii) All options expire in six months.

(iii) The continuously compounded risk-free interest rate is 10%.

You are interested in the graph for the price of an option as a function of the current stock price. In each of the following four charts I - IV, the horizontal axis, S, represents the current stock price, and the vertical axis, π, represents the price of an option.



Match the option with the shaded region in which its graph lies.

If there are two or more possibilities, choose the chart with the smallest shaded region.

	European Call	American Call	European Put	American Put
(A)	I	I	III	III
(B)	II	I	IV	III
(C)	II	I	III	III
(D)	II	II	IV	III
(E)	II	II	IV	IV

10.25 (CAS3, 5/07, Q.12) (2.5 points)
Which of the following effects are correct on the price of a stock option?
I. The premiums would not decrease if the options were American rather than European.
II. For European put, the premiums increase when the stock price increases.
III. For American call, the premiums increase when the strike price increases.
A. I only B. I and II only C. I and III only D. II and III only E. I, II and III

10.26 (CAS3, 11/07, Q.13) (2.5 points) Given the following chart about call options on a particular dividend paying stock, which option has the highest value?

Option	Option Style	Time Until Expiration	Strike Price	Stock Price
A	European	1 year	50	42
B	American	1 year	50	42
C	European	2 years	50	42
D	American	2 years	50	42
E	American	2 years	55	42

A. Option A B. Option B C. Option C D. Option D E. Option E