

Mahler's Guide to

**“Fundamentals of
Individual Risk Rating”
by William R. Gillam
and Richard H. Snader**

Problems

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Solutions to problems are in study guide C.



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Study Aid F10-9-**1B**

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Problems:

1. (14 points) Use the following Workers Compensation data on 20 risks, each with \$500,000 in Standard Premium. Show all work.

Risk	Large Accidents (\$000)				Other Losses (\$000)	Total Losses (\$000)	Loss Ratio	Entry Ratio
A					80	80	16%	0.20
B					120	120	24%	0.30
C					160	160	32%	0.40
D					200	200	40%	0.50
E					200	200	40%	0.50
F	175				25	200	40%	0.50
G					240	240	48%	0.60
H	175				65	240	48%	0.60
I	125				155	280	56%	0.70
J	200				80	280	56%	0.70
K					280	280	56%	0.70
L	300				60	360	72%	0.90
M	125				355	480	96%	1.20
N	125	125	150		80	480	96%	1.20
O					520	520	104%	1.30
P	125	250			185	560	112%	1.40
Q	500				220	720	144%	1.80
R	125	150			525	800	160%	2.00
S	125	615			140	880	176%	2.20
T	150	150	150	175	295	920	184%	2.30
Sum					3985	8000	80%	

For each risk, only accidents larger than \$100,000 are shown.

- a. (4 points) Construct a Table M of insurance charges and savings, for 0.1 increments in entry ratios from 0 to 2.3.
- b. (3 points) Using this Table M, calculate the Basic Premium for the following Retrospective Rating Plan: Maximum Premium = G = 150%, Minimum Premium H = 50%, Loss Conversion Factor = c = 1.05, Expenses = e = 10%, Tax Multiplier = T = 1.02.
- c. (2 points) Calculate the indicated Excess Loss Factor (ELF), for an accident limit of \$250,000.
- d. (1 point) Assume that for the retrospective rating plan in part (b), a \$250,000 accident limit were selected. Using ICRL what would be the losses used for selecting the Expected Loss Group?
- e. (1 points) What are the two “balance” equations one would use in order to determine the Basic Premium for the retrospective rating plan in part (d)?
- f. (3 points) Use the given data to determine the equation for the premium for the following Retrospective Rating Plan:
 Maximum Loss Ratio = 120%, Minimum Loss Ratio = 50%, Accident Limit \$100,000,
 Loss Conversion Factor = 1.05, Expenses = e = 10%, Tax Multiplier = 1.02.

2. (2 points) Credibility has three criteria that must be met. List the three criteria.

Show that each of these criteria is met by the formula for credibility used in the Insurance Services Office (ISO) Commercial General Liability Experience and Schedule Rating Plan, $Z = E/(E + K)$.

3. (1 point) Aggregate Losses are Pareto distributed with shape parameter of 3.

$$F(x) = 1 - \frac{1}{(1 + x/\theta)^3}. \text{ Calculate } \phi(1) \text{ and } \phi(2).$$

4. (3 points) Let the credibility be given by: $Z = (E + I)/(J E + K)$, where E is the size of risk, and I, J, and K are positive constants that do not depend on E.

Determine what if any additional conditions must hold in order to satisfy each of the three criteria given by Gillam and Snader.

5. (1 point) Aggregate Losses are exponentially distributed. Calculate $\phi(1)$ and $\phi(2)$.

6. (1 point) The loss group adjustment factor is 1.6.

The expected loss ratio is 72%.

What is the Excess Loss Factor?

7. (9, 11/86, Q.32) (3 points)

The following Table M charges were constructed from a group of 20 identically sized insureds.

The mean loss ratio is 0.5 and all loss ratios are integral multiples of 0.05.

<u>Loss ratio</u>	<u>Table M charge</u>
0	1
0.05	0.9
0.1	0.8
0.15	0.71
0.2	0.635
0.3	0.5
0.4	0.385
0.5	0.29
0.6	0.22
0.7	0.16

a. What is the savings for a loss ratio of 0.6?

b. What was the smallest loss ratio in the group?

c. Based on the information given above, what is the highest possible loss ratio in the group?

8. (2 points) In the previous question, 9, 11/86, Q.32, determine the number of risks at each loss ratio.

9. (9, 11/92, Q.37) (2 points) You are given the following information pertaining to a particular workers compensation classification:

Full coverage pure premium rate (excluding all loss adjustment expense) = \$1.25.

Medical pure premium rate adjustment (excluding all loss adjustment expense) = \$0.50.

Breakdown of full coverage manual premium rate:

Expected loss ratio	60%
Loss adjustment expense provision	9%
Acquisition expense provision	15%
General expense provision	8%
Tax provision	3%
Profit provision	5%
TOTAL	100%

a. Based on the Gillam and Snader's "Fundamentals of Individual Risk Rating," calculate the ex-medical discount.

Assume that election of ex-medical coverage eliminates 80% of the medical pure premium.

b. Gillam and Snader also discuss an adjustment which should be made to the loss conversion factor when ex-medical coverage is rated retrospectively.

If the full coverage loss conversion factor is 1.15, what is the indicated ex-medical loss conversion factor?

10. (9, 11/96, Q.5) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following are true about credibility as defined in the Revised 1991 Workers Compensation Experience Rating Plan?

1. No risk is so large that its primary losses are fully credible.
2. Only the smallest risks have excess credibility of zero.
3. Primary losses are effectively given more weight in the modification formula than in the prior rating plan.

- A. 1 B. 3 C. 1, 2 D. 1, 3 E. 2, 3

11. (9, 11/96, Q.10) (1 point) Given the following four risks subject to retrospective rating, what is the savings for the maximum premium, if the maximum premium is produced by an 80% loss ratio?

<u>Risk #</u>	<u>Loss Ratio</u>	<u>Standard Premium</u>
1	30%	\$10,000
2	50%	\$10,000
3	60%	\$10,000
4	60%	\$10,000

12. (9, 11/96, Q.11) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following considerations indicate that a small part of the medical pure premium should be retained when pricing workers compensation ex-medical coverage?

1. The insurer retains an obligation to pay medical loss in the event the employer is unable to pay.
 2. Selection of ex-medical coverage will likely be adverse to the insured.
 3. Payment of certain medical costs may be required even though the policy may be ex-medical.
- A. 1 B. 2 C. 1, 3 D. 2, 3 E. 1, 2, 3

13. (9, 11/96, Q.29) (3 points) You are hired as a consultant to a workers compensation risk pool whose members were switched from mandatory guaranteed cost program to a flexible retrospective rating program. The switch occurred three years ago.

The pool's management has notified you of the following three facts:

- In the past three years, many members have been changing their minimum and maximum premium factors and loss limitation with each renewal.
- The state courts have overturned several workers compensation reform initiatives that were expected to save insurers a significant amount in loss costs.
- The pool's underwriting results have deteriorated substantially over the last three years.

The pool has asked you to comment on the possible reasons for their poor results.

- a. (1 point) Give one reason based on Gillam and Snader's "Fundamentals of Individual Risk Rating."
- b. (1 point) Recommend one step the pool should take to improve its underwriting results, assuming it remains on a flexible retrospective rating program. Confine your answer to issues specifically related to retrospective rating.
- c. (1 point) Would you recommend that the pool goes back to a guaranteed cost program on a mandatory basis? Why or why not?

Comment: This past exam question has been rewritten in order to match the current syllabus.

14. (9, 11/96, Q.31) (2 points) Calculate the expense and contingency provision (excluding taxes) in the basic premium factor for a retrospectively rated policy, given the following information.

Show all work.

Minimum Premium Factor	70.0%
Maximum Premium Factor	150.0%
Loss Conversion Factor	1.07
Tax Multiplier	1.04
Expected Loss Ratio	53.4%
Expense and Contingency Ratio (Including Taxes)	35.0%
Net Insurance Charge	13.4%

Note: This past exam question has been rewritten to match the current syllabus.

15. (9, 11/96, Q.32) (4 points) Gillam and Snader describe a system of balance equations in "Fundamentals of Individual Risk Rating."

- a. (1 point) Using the notation in the Gillam and Snader paper, write these equations and define each of the variables in the equations.
- b. (1 point) Describe how these equations are used in practice when Table M is available.
- c. (2 points) Lee, in "The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach," states that one may interpret the charge difference to be the difference between the expected retrospective premium and the minimum premium, apart from the loss conversion factor.

Using a graph and the notation from part a, show both the expected retrospective premium and the minimum premium (ignoring the loss conversion factor), and prove that Lee's statement is true. Show all work.

16. (9, 11/96, Q.34) (2 points) A workers compensation risk pool has 100 risks of exactly the same size. These risks have been retrospectively rated, but the pool's management has decided to move to a guaranteed cost program. They have given you the following information about their current program, and they want you to complete the premium discount table below.

<u>Standard Premium</u>	<u>Discount</u>
First \$5,000	0%
Next \$45,000	???
Next \$50,000	6%
Next \$100,000	9%

Ratios to Standard Premium

Basic Premium Factor	0.29
Converted Insurance Charge	0.05
Loss Conversion Factor	1.10
Tax Multiplier	1.05

- Expenses and profit, excluding taxes are equal to 50% of expected losses.
- Each risk has \$125,000 in standard premium.

17. (9, 11/96, Q.36) (3 points) Your company has traditionally sold homeowners insurance using a \$100 straight deductible. Your company president has asked you to price a disappearing deductible with a retention of \$200 and a limit of \$1,000.

You have been given the following information:

Claim Size Range Net of \$100 Deductible	Count	Total Amount Net of the \$100 Deductible
\$0 - \$100	600	\$36,000
\$101 - \$200	500	\$75,000
\$201 - \$900	750	\$375,000
\$901 - \$1,000	50	\$46,000
\$1,001 - Over	100	\$200,000

	Percent of Premium Reflecting the \$100 Deductible
Allocated Loss Adjustment Expense (ALAE)	5%
Expected Loss Ratio including ALAE	65%
Acquisition Costs	15%
Tax Provision	5%
Underwriting and Claim Overhead	10%
Profit	5%
Safety Factor	95%

- a. (2 points) Calculate the percentage of current retained company losses which would be eliminated by moving from the current \$100 straight deductible to the option proposed by your company president.
- b. (1 point) Calculate the deductible credit (expressed as a percentage of the \$100 straight deductible premium) based on the change mentioned in part a.

Note: Part c of this exam question involved franchise deductibles, no longer of the syllabus.

18. (9, 11/97, Q.1) (1 point) A Workers Compensation risk has been retrospectively rated in the past but is contemplating moving to a guaranteed cost basis.

The following information pertains to the risk:

Standard Premium	\$350,000
Basic Premium Factor	0.284
Converted Insurance Charge	0.120
Loss Conversion Factor	1.080
Tax Multiplier	1.035
Expenses and Profit (excluding taxes)	0.220

Calculate the guaranteed cost premium.

19. (9, 11/97, Q.12) (1 point)

According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following reasons contributed to the growth in interest in higher deductible options?

1. There is a positive cash flow to the insurer, who can retain the premium until excess claims have to be paid.
2. Many companies cannot obtain the regulator's approval to self-insure their risk, so they will purchase the deductible option.
3. In lines with residual market pools, deductible plans will reduce the state assessments to the insurance carrier.

A. 1, 2 B. 1, 3 C. 2, 3 D. 1, 2, 3 E. None of 1, 2, 3 are true.

20. (9, 11/97, Q.13) (1 point) Given a homeowner's policy with a disappearing deductible with lower bound of \$100 and upper bound of \$500, calculate the loss elimination ratio if the following claims are presented by the homeowner: \$75, \$125, \$225, and \$530.

21. (9, 11/97, Q.21) (2 points)

In "Fundamentals of Individual Risk Rating," Gillam and Snader define the tax multiplier.

a. (1 point) Calculate the tax multiplier given the following information. Show all work.

Expected Loss Ratio	65%
Premium Tax	3%
Second Injury Fund Assessment	1%

b. (1 point) What is the purpose of the constant factor in the formula for the tax multiplier?

The following information should be used to answer questions 9, 11/97, #22 and #23.

Number of Risks @			
<u>Loss Ratio</u>	<u>Loss Ratio</u>	<u>Entry Ratio</u>	<u>Excess Ratio</u>
0%	0	0.000	1.000
10%	1	0.169	0.831
20%	2	0.339	0.678
30%	0	0.508	0.559
40%	1	0.679	0.441
50%	0	0.847	0.339
60%	1	1.017	0.237
70%	1	1.186	0.153
80%	1	1.356	0.085
90%	2	1.525	0.034
100%	0	1.695	0.017
110%	1	1.864	0.000
120%	0	2.034	0.000

The following information is for each risk:

Standard Premium	\$100,000
Provision for Losses and Expenses Exclusive of Taxes	\$96,000
Loss Conversion Factor	1.20
Selected Maximum Loss Ratio	90%
Selected Minimum Loss Ratio	20%
Tax Multiplier	1.00

22. (9, 11/97, Q.22) (3 points) Using Gillam and Snader's "Fundamentals of Individual Risk Rating," demonstrate the concept of actuarial balance between retrospective and prospective rating. Show all work.

23. (9, 11/97, Q.23) Use the information on the previous page to answer the following scenario: Your underwriter, Terry Ball, comes to inform you that 2 of the 10 insureds have not renewed their policies. According to your calculations, these two insureds should have actual, developed loss ratios of 20% and 80%, respectively, for the latest policy year.

Another underwriter, Max Exciting, comes to you with a new prospect with a standard premium of \$100,000, a loss and expense provision (excluding taxes) of \$96,000, and an expected loss ratio of 50%. Because the 50% loss ratio equals the average of the actual, developed loss ratios of the two non-renewed insureds, the underwriter wants to charge the new prospect a guaranteed cost premium equal to the average of the two non-renewed insureds' retrospective premiums.

- a. (1 point) Give one reason why you should disagree with Max Exciting.
- b. (2 points) Reconstruct Table M using the loss ratio data for the 8 retrospectively rated insureds who have renewed their policies. Show all work.
- c. (2 points) Due to the soft market, you have to lower the loss conversion factor to 1.15 for the 8 insureds in part (b). However, you decide to use the same basic premium factor as when you had 10 insureds. Show how you can accomplish this, while maintaining an actuarially balanced retrospective rating plan. In addition, calculate the guaranteed cost premium you would charge each insured individually. Show all work.

24. (9, 11/97, Q.27) (2 points)

- a. (1 point) Briefly describe the Insurance Charge Reflecting Loss Limitation (ICRLL) procedure.
- b. (1 point) Give one advantage and one disadvantage of the ICRLL procedure.

25. (9, 11/97, Q.31) (2.5 points)

- a. (0.5 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," what does the excess element of a split plan reflect? How much weight is given to the excess element relative to the primary element in an experience rating formula?
- b. (1 point) A large risk has instituted a 'return to work' program. How would this risk react to the change in excess credibility put in place with the 1991 Revised Experience Rating Plan?
- c. (1 point) Another large risk has hired a nurse to handle all the medical only claims at its main production plant instead of reporting them to the insurance company. How would this risk react to the change in credibility put in place with the 1991 Revised Experience Rating Plan?

Note: I have rewritten part c of this past exam question.

26. (9, 11/97, Q.32) (2 points) Using Gillam and Snader's "Fundamentals of Individual Risk Rating," state Perryman's First Formula and derive a formula for the primary credibility as a function of the expected losses, the weighting factor W , and the constant K . Show all work.

27. (9, 11/97, Q.35a) (2 points) You are the consulting actuary for a group of actuarial professionals who pool their actuarial malpractice risk in a captive. The captive offers a \$10 million policy limit to each professional in 1997. In 1998, a commercial insurer will start to write actuarial malpractice policies at a \$1 million limit and all pool members have decided that they will buy their first \$1 million policy from the new market. The pool will offer a \$9 million policy in excess of the \$1 million limit provided by the primary insurer.

You are given the following information:

- The portion of the allocated loss adjustment expense eliminated is proportional to the losses eliminated.
- Acquisition expenses and taxes vary with premiums at 10% and 5% respectively.
 These provisions apply to both the \$10 million limit full coverage and the \$9 million excess of \$1 million policies written by the captive. No profit or risk load is assessed.
- The remainder of the expenses are the captive management fees, which are a fixed portion of the full coverage premium at 5%.
- The pool's board has recommended a safety factor of 90%.
- The total premium for the full \$10 million limit is \$7 million.
- The following excess loss distribution applies to professional malpractice for actuaries:

<u>Limit</u>	<u>Excess Loss Distribution as a Percent of Total Losses</u>
1,000,000	68.5%
10,000,000	18.5%

Calculate the total premium for the new \$9 million excess of \$1 million coverage. Show all work.

28. (9, 11/98, Q.4) (1 point) Which of the following are usually included in the basic premium?

1. Cost of the insurance carrier's claim services.
2. An adjustment for limiting the retrospective premium between the minimum retrospective premium and the maximum retrospective premium.
3. Loss control services.

Note: I have slightly rewritten this past exam question in order to match the current syllabus.

- A. 1 B. 2 C. 1, 3 D. 2, 3 E. 1, 2, 3

29. (9, 11/98, Q.6) (1 point) Using Gillam and Snader's "Fundamentals of Individual Risk Rating" and given the following information, what is the loss conversion factor for the ex-medical coverage?

Full Coverage Pure Premium Rate (not including LAE): \$1.75

Medical Pure Premium Rate (not including LAE): \$0.75

Full Coverage Loss Conversion Factor: 1.15

Full Coverage Expected Loss Ratio: 65%

Full Coverage Provision for Taxes: 2%

Full Coverage Provision for Acquisition: 13%

Full Coverage Provision for Profit: 5%

Full Coverage Provision for General Expenses: 6%

Full Coverage Provision for LAE: 9%

Assume that the election of ex-medical coverage eliminates 80% of the medical pure premium.

30. (9, 11/98, Q.16) (1 point) You are calculating the experience modification for an insured.

For one of the claims, the primary actual loss subject to experience rating is \$6,000, and the excess actual loss subject to experience rating is \$4,000. According to the 1961 Workers Compensation Experience Rating Plan as described in Gillam and Snader's "Fundamentals of Individual Risk Rating," how much of the claim (in dollars) is being eliminated by the maximum ratable value?

Assume that the parameter I -- the value below which the primary actual losses equal the total actual losses -- is \$2,000, and assume that C -- the constant parameter -- is \$8,000.

31. (9, 11/98, Q.31) (3 points) You are the actuary for Whatever Insurance Company (WIC), based in the Republic of Pandas. Under its automobile collision coverage, WIC has been using a \$250 deductible disappearing at \$750. This structure has caused confusion among WIC brokers and underwriters, and your president has asked you to derive a simpler approach that will have no impact on WIC's current rate. You suggest a straight deductible method.

a. (2.5 points) Based on the ground-up size-of-loss distribution shown below, what straight deductible should WIC use if it wants the change in deductible structure to have no impact on its current rates? Assume that expenses, taxes, and profit and contingency loadings are held constant. Show all work.

(Hint: The solution is an even multiple of \$25, e.g., \$0, \$25, \$50, etc.)

<u>Size of Loss</u>	<u>Number of Claims</u>	<u>Total Losses</u>
\$25	8	\$200
\$50	19	\$950
\$100	23	\$2,300
\$200	38	\$7,600
\$250	44	\$11,000
\$300	39	\$11,700
\$500	21	\$10,500
\$750	7	\$5,250
\$1,000	1	\$1,000
Totals	200	\$50,500

b. (0.5 point) What is the expected average loss payment of claims exceeding the new straight deductible determined in part a. ?

(In other words, what is the average payment per payment.)

32. (9, 11/98, Q.37) (2.5 points) Use Gillam and Snader's "Fundamentals of Individual Risk Rating" and the National Council on Compensation Insurance's Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance to answer these questions.

An insured is expected to have five claims, resulting in total expected losses of \$15,000. However, the insured only has one claim for \$12,000. Assume that the discount ratio (D) is 0.2, the weighting value (W) is 0.5, and there is no maximum ratable value.

a. (1.25 points) According to the 1961 Workers Compensation Experience Rating Plan, will this insured receive a credit modification? Assume that the constant (C) is \$8,000 and the split point (I) is \$2,000. Show all work.

b. (1.25 points) According to the 1991 Revised Workers Compensation Experience Rating Plan, will this insured receive a credit modification? In answering this question, assume that the D and W values for this risk under the 1991 Plan are the same as those under the 1961 Plan. Show all work.

33. (9, 11/98, Q.38) (1.5 points) Given the following information on a Workers Compensation risk, calculate the risk's expected losses. Show all work.

Loss free credit = 25%

Experience modification = 0.80

Actual losses = \$50,000

The risk is rated under a no split plan.

34. (9, 11/98, Q.39) (3.25 points)

Use Gillam and Snader's "Fundamentals of Individual Risk Rating" to answer the following:

- a. (0.75 point) In mathematical terms, give the three criteria for proper credibility.
- b. (0.5 point) Compare the experience rating plan credibility values for two risks with expected losses of \$100,000 and \$500,000, respectively. Assume a no split plan.
- c. (0.5 point) Compare the change in the modification factor due to an additional \$50,000 claim for two risks with expected losses of \$100,000 and \$500,000, respectively. Assume a no split plan.
- d. (1.5 points) Which of the three criteria from part a. does the following credibility formula satisfy?
 $Z = [E + K] / [JE + K]$, where $J > 1$ and $K > 0$. Show all work.

35. (9, 11/98, Q.44c) (1.5 points)

You are given the following information for a retrospective rating plan:

Standard Premium	\$1,000,000
Maximum Premium	\$1,350,000
Minimum Premium	\$550,000
Tax Multiplier	1.05
Loss Conversion Factor	1.10
Unlimited Expected Loss Ratio	62.0%
Limited Expected Loss Ratio	31.0%
Expense and Profit (exclusive of taxes)	20.0%
State / Hazard Group Severity Multiplier	1.05

Table of Insurance Charges

<u>Entry Ratio</u>	<u>Expected Loss Range</u> <u>\$575,000 - \$1,500,000</u>	<u>Expected Loss Range</u> <u>\$1,500,001 - \$2,000,000</u>
0.03	0.98	0.98
0.07	0.94	0.94
0.08	0.93	0.93
0.11	0.90	0.90
1.20	0.48	0.46
1.23	0.47	0.45
2.26	0.14	0.11
2.30	0.11	0.09

Calculate the losses used to enter Table M and the excess loss premium using the Insurance Charge Reflecting Loss Limitation (ICRLL) procedure. Show all work.

Note: Parts a and b of this past exam question were based on material not on the current syllabus.

36. (9, 11/98, Q.47) (3 points) Using Gillam and Snader's "Fundamentals of Individual Risk Rating" and the information provided, answer the following questions.

<u>Loss Ratio</u>	<u>Number of Risks at Loss Ratio</u>
0%	0
20%	1
40%	1
60%	3
80%	1
100%	1
120%	0
Total	7

Standard Premium of Each Risk \$10,000

Maximum Loss Ratio 80%

Minimum Loss Ratio 40%

- (1 point) What is the excess pure premium ratio at the 60% loss ratio? Show all work.
- (1 point) What is the entry ratio corresponding to the minimum loss ratio? Show all work.
- (1 point) Calculate S_G , the savings at the maximum. Show all work.

37. (9, 11/99, Q.6) (1 point) Which of the following are true with regard to the insurance charge reflecting loss limitation procedure?

- The procedure requires entry ratios based on expected unlimited losses.
- The procedure requires the recalculation of the insurance charge for pricing each loss limit.
- The losses used to select the Table M column are the product of expected unlimited losses, a state/hazard group severity multiplier, and a factor to reflect the selected loss limit.

A. 1 B. 2 C. 1, 3 D. 2, 3 E. 1, 2, 3

Note: I have rewritten this past exam question to match the current syllabus.

38. (9, 11/99, Q.12) (1 point) Using the Workers Compensation split plan as described in Gillam and Snader's "Fundamentals of Individual Risk Rating" and the following information, what is the experience modification?

Expected Loss	\$500
Expected Excess Loss	\$200
Primary Credibility	0.5
Excess Credibility	0.4
Primary Actual Loss	\$600
Excess Actual Loss	\$200

39. (9, 11/99, Q.13) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," the basic formula for experience rating does not work well in Workers Compensation because:

- A. The loss distribution is heavy tailed.
- B. The loss distribution is skewed to the left.
- C. Losses do not include allocated loss adjustment expenses.
- D. Calendar year financial data is used in the calculation of the modification.
- E. Workers Compensation Insurance is mandatory in most states.

Note: I have rewritten this past exam question.

40. (9, 11/99, Q.21) (2.75 points) Using Gillam and Snader's "Fundamentals of Individual Risk Rating," derive the charge difference and ratio difference equations by starting with the formula for the maximum and/or minimum retrospective premium. Define all notation and show all work.

41. (9, 11/99, Q.41) (2 points) You are the pricing actuary for Wasabi Insurance. You have determined that the discount that would result from the introduction of a \$50 deductible disappearing at \$500 is 13%.

Based on Gillam and Snader's "Fundamentals of Individual Risk Rating" and on the information provided below, what safety factor have you used in your calculations? Show all work.

<u>Size of Loss (\$)</u>	<u>Number of Claims</u>	<u>Total Losses (\$)</u>
25	60	1,500
50	108	5,400
100	111	11,100
250	66	16,500
500	34	17,000
1,000	21	21,000
Total	400	72,500

The adequate full coverage premium is \$262.50. The provision for acquisition expenses, taxes, and profit amounts to 45% of the premium. The provision for all other expenses (including allocated loss adjustment expense) amounts to 15% of the full coverage premium.

42. (9, 11/00, Q.4) (1 point) Using the split plan formula from Gillam and Snader's "Fundamentals of Individual Risk Rating" and the following data, calculate the experience modification.

- Z_p .500
- Z_e .400
- E_e \$2,000
- E \$5,000
- A_p \$6,000
- A \$7,500

43. (9, 11/00, Q.5) (1 point) You are given the following information:

Standard Premium	\$100,000
Expected Losses	\$60,000
Expected Losses Reflecting an Occurrence Loss Limitation	\$54,000

Based on Gillam and Snader's "Fundamentals of Individual Risk Rating," calculate the adjusted expected losses used to determine the appropriate Table M column.

44. (9, 11/00, Q.6) (1 point) You are given the following information:

Payroll	\$25,000,000
Rate per \$100 of Payroll	1.500
Experience Rating Modification	0.900
Tax Multiplier	1.050

Stock Carrier Discount Table

First \$5,000	0.0%
Next \$95,000	10.9%
Next \$400,000	12.6%
Over 5500,000	14.4%

Based on Gillam and Snader's "Fundamentals of Individual Risk Rating," calculate the guaranteed cost premium.

45. (9, 11/00, Q.20) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following are true?

1. A safety factor is used in determining the discount for deductibles.
2. For deductible coverage for liability insurance, a portion of loss adjustment expenses is eliminated along with the rest of the losses.
3. When the ex-medical discount is derived, the portion of medical pure premium eliminated is determined by judgment.

A. 1 B. 3 C. 1, 2 D. 1, 3 E. 2, 3

Note: I have rewritten this past exam question.

46. (9, 11/00, Q.40) (3 points) In Gillam and Snader's "Fundamentals of Individual Risk Rating," the authors describe three criteria for an effective credibility function in both qualitative and mathematical terms. Use Gillam and Snader to answer the following.

- a. (1 point) Briefly describe in words the authors' three criteria.
- b. (1 point) Show the three criteria in formulas. Define all notation used.
- c. (1 point) For each of the three criteria, determine whether or not the following credibility function satisfies that criterion. If it does not, briefly explain why.

<u>Expected Losses Between</u>	<u>Credibility</u>
\$0 - \$5,000	0%
\$5,001 - \$50,000	50%
\$50,001 or more	100%

47. (9, 11/00, Q.44) (2 points) An insured is deciding whether or not to be retrospectively rated. The insured is given the following information about a possible retrospectively rated program:

Standard Premium	\$100,000
Basic Premium	\$20,000
Expected Losses	\$60,000
Converted Insurance Charge	\$1,950
Loss Conversion Factor	1.300
Taxes as a Percent of Premium	2%

Based on Gillam and Snader's "Fundamentals of Individual Risk Rating," answer the following questions.

- a. (1.5 points) What premium will the insured pay if he decides not to be retrospectively rated?
Show all work.
- b. (0.5 point) At what level of incurred losses will the insured pay the same premium whether or not he decides to be retrospectively rated? Show all work.

48. (9, 11/00, Q.48) (3 points) Based on Gillam and Snader's "Fundamentals of Individual Risk Rating," and the data below, answer the following. Assume that all risks have equal standard premium. Show all work.

<u>Number of Risks</u>	<u>Unlimited Loss Ratio</u>
1	10%
4	30%
2	40%
4	50%
1	60%
3	70%
1	80%
2	100%
2	120%

<u>Number of Risks</u>	<u>Limited Loss Ratio</u>
1	10%
4	30%
4	40%
3	50%
2	60%
1	70%
2	80%
2	90%
1	110%

- (0.5 point) Calculate the unlimited expected loss ratio and the limited expected loss ratio.
- (0.5 point) Calculate the loss elimination ratio.
- (1.5 points) Calculate the Table M charges at loss ratios from 0% to 120% using loss ratio increments of 10%.
- (0.5 point) Calculate the Table M savings for an entry ratio of 0.50.

49. (9, 11/01, Q.8) (1 point) Based on Gillam and Snader's "Fundamentals of Individual Risk Rating" and the information for a retrospectively rated policy in the table below, determine the guaranteed cost premium.

Standard Premium	\$100,000
Loss Conversion Factor	1.10
Expected Loss Ratio	0.60
Tax Multiplier	1.03
Expenses and Profit, Excluding Taxes	0.20
Basic Premium Factor	0.30

50. (9, 11/01, Q.10) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following statements are true?

1. If the minimum premium is increased but the maximum premium and the loss conversion factor remain the same, then the basic premium will necessarily increase.
 2. If the maximum premium is decreased but the minimum premium is increased by an equal amount, then the basic premium will necessarily remain the same.
 3. If the basic premium and loss conversion factor remain the same but the minimum premium increases, then the maximum premium will necessarily decrease.
- A. 1 B. 2 C. 3 D. 1, 3 E. 2, 3

51. (9, 11/01, Q.13) (1 point) According to Gillam and Snader's "Fundamentals of Individual Risk Rating," which of the following will increase the discount for excess coverage?

1. An increase in the expected loss ratio.
 2. An increase in the portion of the loss elimination ratio that is expected to be realized.
 3. An increase in the portion of expenses that vary with full coverage premiums.
- A. 1 B. 1, 2 C. 1, 3 D. 2, 3 E. 1, 2, 3

52. (9, 11/01, Q.31) (2 points) Using Gillam and Snader's "Fundamentals of Individual Risk Rating" and the information below, answer the following questions. Show all work.

Standard Premium	\$400,000
Guaranteed Cost Premium	\$370,000
Loss Conversion Factor	1.10
Tax Multiplier	1.05
Expected Loss Ratio	0.50
Basic Premium Factor	0.37
Entry Ratio at Maximum Premium	1.00

Table of Insurance Charges

<u>Expected Loss Group</u>	<u>Expected Loss Range</u>	<u>Expenses and Profit Excluding Taxes</u>	<u>Insurance Charge at 1.00 Entry Ratio</u>
39	\$210,469 - \$227,507	0.184	0.39
40	\$194,842 - \$210,468	0.200	0.40
41	\$180,486 - \$194,841	0.215	0.41

- a. (1 point) Calculate the savings at the minimum.
- b. (1 point) Calculate the minimum premium factor which is consistent with the basic premium factor.

53. (9, 11/01, Q.32) (3 points)

Based on Gillam and Snader's "Fundamentals of Individual Risk Rating," answer the following.

- a. (1.5 points) You are the risk manager for a large company that buys a retrospectively rated Workers Compensation policy. This policy was rated using an expected loss ratio of 60%. Your boss asks you to determine how the final retrospectively rated premium for this policy will compare to the guaranteed cost premium if the loss ratio at the final evaluation is equal to 60%. Briefly describe how the final retrospectively rated premium and the guaranteed cost premium will compare. Assume the maximum premium factor corresponds to a loss ratio higher than 60%, and the minimum premium factor corresponds to a loss ratio lower than 60%.
- b. (1.5 points) Your boss is suspicious of the complicated trial-and-error procedure used by the insurer to calculate an insurance charge for your company's policy. Briefly explain why such an iterative procedure is used.

54. (9, 11/01, Q.36) (2 points) You have been asked to develop a discount for use in Commercial Auto Personal Injury Protection (PIP) insurance in the state of Rocky Mount. The legislature has mandated that a discount be offered to insureds that opt for a 20% co-payment on all medical losses covered by PIP.

Using Gillam and Snader's "Fundamentals of Individual Risk Rating" and the information below, calculate the appropriate discount for insureds that opt for the 20% co-payment.

Show all work.

Distribution of Losses Covered by PIP

Medical Losses	60%
Loss of Income Losses	40%

	<u>As a Ratio to</u>	<u>Percentage</u>
Acquisition Expense	Premium	15.0%
Taxes	Premium	3.0%
Profit Provision	Premium	4.0%
Allocated Loss Adjustment Expenses (ALAE)	Losses	10.0%
Expected Loss Ratio (including ALAE)	Premium	66.0%
All Other Expenses	Premium	12.0%

Safety Factor 90%.

55. (9, 11/02, Q.2) (1 point) Which of the following are considerations for retaining a part of the medical pure premium in calculating the workers compensation ex-medical discount?

1. Selection of ex-medical coverage will likely be adverse to the insurer.
2. Payment of some medical costs may be required even though the policy may be ex-medical.
3. The insurer retains an obligation to pay medical loss in the event the employer is unable to pay.

- A. 2 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. 1, 2, and 3

56. (9, 11/02, Q.14) (1 point) For entry ratios r , with a cumulative distribution function F and density f , the excess pure premium ratio X_G can be represented as

$$1. X_G = \int_{r_G}^{\infty} (r - r_G) f(r) dr / \int_0^{\infty} r f(r) dr$$

$$2. X_G = \int_{r_G}^{\infty} (r - r_G) f(r) dr$$

$$3. X_G = \int_0^{r_G} (r_G - r) f(r) dr$$

A. 3 only B. 1 and 2 only C. 1 and 3 only D. 2 and 3 only E. 1, 2, and 3

57. (9, 11/02, Q.15) (1 point) Calculate the Table of Insurance Charges value difference for a retrospective policy based on the rating factors provided.

A loss limit of \$250,000 has been selected for this policy.

E	0.70
e	0.30
G	1.50
H	0.50
T	1.05
c	1.10
ELF at \$250,000 (F)	0.10

58. (9, 11/02, Q.16) (1 point) Given the information below for a retrospectively rated policy, calculate the net converted insurance charge.

Standard Premium	\$1,000,000
Loss Conversion Factor	1.1
Expected Loss Ratio	0.6
Maximum Loss Ratio	1.5
Minimum Loss Ratio	0.3
Table M Charge at Maximum	0.084
Table M Charge at Minimum	0.551

59. (9, 11/02, Q.36) (2.5 points) A hypothetical Table M is constructed based on the experience of the following 10 risks, each with standard premium of \$100,000.

<u>Risk</u>	<u>Loss Ratio</u>
1	10%
2	60%
3	0%
4	80%
5	60%
6	60%
7	120%
8	100%
9	70%
10	40%

Assume a hypothetical retrospective rating contract based on this Table M, with a maximum premium produced by a 90% loss ratio and a minimum premium produced by a 20% loss ratio. The loss conversion factor is 1.10.

Calculate the net converted insurance charge.

60. (9, 11/03, Q.7) (1 point) Given the following information, calculate the tax multiplier.

Premium Tax Rate	4.0%
Insolvency Fund Loss Based Assessment	1.0%
Second Injury Fund Loss Based Assessment	1.5%
Permissible Loss Ratio	60.0%

61. (9, 11/03, Q.10) (1 point)

Given the information below for a retrospectively rated policy, calculate the basic premium.

Standard Premium	1,000,000
Loss Conversion Factor	1.1
Expected Loss Ratio	0.7
Total Expenses, excluding taxes, as a ratio of Standard Premium	0.25
Table M Charge at Maximum	0.057
Table M Savings at Minimum	0.041

62. (9, 11/03, Q.28) (3 points)

a. (1.5 points) In "Fundamentals of Individual Risk Rating," Gillam and Snader provide the following transformed equation for the basic experience rating formula:

$$M = 1 + Z (A - E)/E.$$

Give a verbal explanation of this formula.

b. (1.5 points) State either algebraically or verbally the conditions credibility should satisfy for the credibility system to operate effectively in experience rating.

63. (9, 11/03, Q.32) (2 points)

Explain why the Table of Insurance Charges in the NCCI Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance varies by expected loss group.

64. (9, 11/03, Q.33) (3 points) You are helping an underwriter price a retrospectively-rated workers compensation policy with a per occurrence limit on losses contributing to the aggregate limit. The policy specifies no minimum aggregate limit. Between last year and this year, the customer has doubled its work force without changing the nature of what it does. Assume that this is the only change between the expiring and renewing policy (i.e. assume no inflation, trends, or benefit changes; also assume no changes to selected limits or deductibles).

How would each of the following change, if at all, due to the increase in exposure? Explain your answers.

- a. (1 point) Premium discount, as a percentage of manual premium.
- b. (1 point) Insurance charge, as a percentage of expected losses.
- c. (1 point) Excess loss pure premium factor (excess losses as a percentage of expected ground-up losses).

65. (9, 11/04, Q.19) (1 point) Given the following loss distribution:

<u>Size of Loss</u>	<u>Number of Claims</u>	<u>Total Loss Dollars</u>
\$100	40	\$4,000
\$250	50	\$12,500
\$500	120	\$60,000
\$750	50	\$37,500
\$1,000 and higher	60	\$86,000
Total	320	\$200,000

What is the loss elimination ratio of a \$250 deductible disappearing at \$750?

66. (9, 11/04, Q.43) (2 points) Given the following sample of loss experience from a collection of five similarly sized risks:

<u>Risk</u>	<u>Annual Loss</u>
1	\$2,100,000
2	\$1,400,000
3	\$800,000
4	\$X
5	\$Y
AVERAGE	\$1,000,000

Calculate the insurance savings $\psi(0.8)$. Show all work.

67. (9, 11/04, Q.47) (2 points) A large employer currently has a retrospectively rated workers compensation policy with the following provisions:

Maximum retrospective premium factor	1.10
Loss conversion factor	1.00
Tax multiplier	1.05

Assume:

- Losses only (excluding ALAE) are subject to retrospective rating.
- Minimum premium factor = basic premium times the tax multiplier.
- No changes in exposure, loss ratio, or underwriting expenses.

List two changes to the current policy (other than exposure, loss ratio, or underwriting expense) that would result in a lower basic premium factor. How would each of these changes achieve the desired result?

68. (5, 5/05, Q.17) (1 point) Which of the functions in the table below define appropriate credibility rules? Assume that full credibility for a claim set is assigned when five or more claims have been observed.

Number of claims	f_1	f_2	f_3
1	0.20	0.42	0.04
2	0.40	0.60	0.16
3	0.60	0.76	0.36
4	0.80	0.89	0.64
5	1.00	1.00	1.00

- A. f_1 only B. f_2 only C. f_1 and f_2 only D. f_1 and f_3 only E. $f_1, f_2,$ and f_3

69. (9, 11/05, Q.30) (1.5 points)

a. (1 point) Given the following information:

Taxes, Licenses, and Fees as % of premium	2.5%
Insolvency fund assessment as a % of loss	1.5%
2 nd injury fund assessment as % of loss	2.0%
Expected loss ratio	50%

Calculate the tax multiplier for a retrospectively rated workers compensation policy effective January 1, 2005.

b. (0.5 points) A client wishes to purchase a retrospectively rated workers compensation policy effective January 1, 2005 with exposures in the following states.

<u>State</u>	<u>Standard Premium</u>
Arkansas	\$4,000,000
Colorado	\$100,000
Virginia	\$600,000

Describe the process for determining the tax multiplier to use in rating the retrospectively rated policy. Do not calculate.

70. (9, 11/05, Q.32) (2 points) Given the following information:

Standard premium	\$750,000
Expected losses	\$375,000
Basic premium factor	0.305
Tax multiplier	1.025
Loss conversion factor	1.050

Premium Discount Table

<u>Premium</u>	<u>Discount</u>
First \$5,000	0.0%
Next \$95,000	10.0%
Next \$400,000	15.0%
Over \$500,000	20.0%

- a. (1.5 points) Would a guaranteed cost policy or a retrospectively rated policy result in a lower expected premium for the insured?
- b. (0.5 points) For a group of risks, why might the premium collected on a group of guaranteed cost policies not be equal to the premium collected on a group of retrospectively rated policies issued to the same group of risks?

71. (9, 11/05, Q.35) (3.5 points) It is determined that a 28.8% credit would result from the introduction of a \$5,000 deductible disappearing at \$15,000.

This credit is based on the following information:

Expected loss ratio (excluding ALAE)	60%
Provision for acquisition, taxes, and profit	25%
Safety factor	90%

<u>Size of Loss</u>	<u>Number of Claims</u>
\$500	1
\$1,000	62
\$5,000	114
\$8,000	120
\$10,000	Y
\$15,000	38
\$20,000	15

Determine the value of Y.

72. (9, 11/06, Q.32) (4 points)

A retrospectively rated workers compensation policy has the following parameters:

Minimum premium factor	0.96
Basic premium factor	0.30
Expected loss ratio	0.60
Expense ratio	0.40
Loss conversion factor	1.05
Tax multiplier	1.00

The insurance charge at entry ratio r is given by the formula:

$$X(r) = 0.8/(0.8 + r).$$

Calculate the maximum premium factor for this policy.

73. (9, 11/06 Q.35) (3 points)

The following apply to a retrospectively rated policy:

Standard Premium	\$950,000
Expected unlimited loss ratio	68%
State/hazard group relativity	95%

Expected loss group after adjusting 28
for the loss limit and state/hazard group

Also, the following table of expected loss ranges applies:

<u>Expected Loss Group</u>	<u>Expected Loss Range</u>
31	\$ 630,000 to \$ 720,000
30	\$ 720,001 to \$ 830,000
29	\$ 830,001 to \$ 990,000
28	\$ 990,001 to \$ 1,180,000
27	\$ 1,180,001 to \$ 1,415,000
26	\$ 1,415,001 to \$ 1,744,000

Calculate the minimum possible value of the excess loss factor.

74. (5, 5/07, Q.38) (1.5 points)

a. (0.75 point) State the criteria for credibility.

b. (0.75 point) Assess whether $F(x) = X^2$, $0 \leq X \leq 1$, meets each of the criteria in part a. above.

Show all work.

75. (9, 11/07, Q.30) (2 points) An actuary is pricing a retrospectively rated policy for which

- The aggregate loss at minimum premium is \$40,000.
- The aggregate loss at maximum premium is \$60,000.

The actuary has decided to base the price of this policy on the following aggregate losses, experienced by 10 policies that are identical to the policy being priced.

<u>Policy Number</u>	<u>Aggregate Losses</u>
1	\$ 5,000
2	\$ 10,000
3	\$ 20,000
4	\$ 40,000
5	\$ 40,000
6	\$ 50,000
7	\$ 60,000
8	\$ 75,000
9	\$ 80,000
10	\$ 120,000

Calculate this policy's net insurance charge as a percent of expected aggregate loss.

76. (9, 11/07, Q.32) (3 points) Five identical workers compensation risks: A, B, C, D, and E, purchased identical retrospectively rated policies with the following characteristics:

Loss Ratio at Minimum Premium:	30%
Loss Ratio at Maximum Premium:	70%
Loss Conversion Factor:	1.200
Converted Insurance Charge:	0.096

Four of these risks experienced the following actual loss ratios:

<u>Risk</u>	<u>Loss Ratio</u>
A	26%
B	45%
C	70%
E	100%

It is known that the loss ratio experienced by risk D is between the loss ratios experienced by risks C and E.

It is also known that the average loss ratio experienced by these 5 risks is equal to the expected loss ratio on which the policies were priced.

Calculate the loss ratio experienced by risk D.

77. (9, 11/07, Q.35) (4 points) Ten identical retrospectively rated workers compensation risks experienced actual losses, as summarized in the table below.

<u>Unlimited Ultimate Loss Ratio</u>	<u>Unlimited Ultimate Losses</u>	<u>Number of Risks</u>
10%	\$ 10,000	1
20%	\$ 20,000	1
60%	\$ 60,000	5
90%	\$ 90,000	3

These policies were priced based on the following information:

Standard Premium	\$100,000
Expected Loss Ratio	60%
Tax Multiplier	1.00
Expense Ratio	36%
Loss Conversion Factor	1.20
Loss Ratio at Minimum Premium	20%
Loss Ratio at Maximum Premium	80%
Table M Charge at Minimum Loss Ratio	0.6833
Table M Charge at Maximum Loss Ratio	0.0833

Demonstrate whether the retrospective rating plan is balanced for these risks to within 1% of total standard premium.

78. (9, 11/09, Q.26) (2.5 points) The following claim count distribution by size of loss is available for a small book of business:

Size of Loss	\$5	\$10	\$15	\$25
Claim Count	50	20	20	10

- a. (1 point) Calculate the loss elimination ratio for a straight \$5 deductible.
- b. (1.5 points) The insurer is considering a disappearing deductible where the \$5 deductible would disappear at \$20. Calculate the resulting loss elimination ratio.

79. (9, 11/09, Q.28) (2 points)

The following applies to a retrospectively rated policy with a \$250,000 loss limitation:

- Minimum premium (H) = \$750,000
- Maximum premium (G) = \$1,500,000
- Expected loss for risk (E) = \$1,000,000
- Provision for expenses in guaranteed cost risk, exclusive of taxes (e) = \$250,000
- Converted insurance charge (cl) = \$150,000
- Tax multiplier (T) = 1.05
- Loss Conversion Factor (c) = 1.1

Individual losses for the policy are:

1. 10,000
2. 300,000
3. 30,000
4. 25,000

Calculate this policy's final retrospective premium.

80. (9, 11/09, Q.31) (3.5 points) The following information applies to a company's retrospectively rated workers compensation policy:

- Its losses are expected to be uniformly distributed between \$0 and \$1,000,000.
 - Losses at maximum premium are \$700,000.
 - Losses at minimum premium are \$200,000.
 - The loss conversion factor is 1.10.
 - The basic premium is \$100,000.
- a. (1.5 points) Calculate the expected retrospective premium for this workers compensation policy.
 - b. (2 points) The company is considering whether to implement a fraud detection device. This addition would result in a shift in the loss distribution to a uniform distribution between \$0 and \$800,000. Assuming that no other plan parameters change, calculate the resulting premium savings.