Mahler's Guide to

Losses

(Chapter 6 of Basic Ratemaking by Werner and Modlin)

Solutions to Problems

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Study Aid S10-5-6C

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Solutions:

1. For 6 month policies the trend from date is: 7/1/10 + (6 months)/2 = 10/1/10.
The average date of writing under the new rates is: 5/1/12 + 6 months = 11/1/12.
The average date of accident is: 11/1/12 + (6 months)/2 = 2/1/13.
The trend period is from 10/1/10 to 2/1/13, or 2 years and four months.
Trended ultimate loss and lae amount for Policy Year 2010 are:
$(1.06)(0.99^{2.333})(1.04^{2.333})(1.28)(\$25 \text{ million})$ = **$36.31 million**.

2. 12,039/132,470 = 9.09%. 13,143/148,466 = 8.85%. 15,286/156,700 = 9.75%.
A reasonable provision would be the average: (9.09% + 8.85% + 9.75%)/3 = **9.2%**.
Comment: See Table 6.18 in Basic Ratemaking.

3. E. 1. Ratemaking usually uses direct data when there is proportional reinsurance, and often uses net data when there is non-proportional reinsurance. 2. The extension of exposures method is more accurate when the appropriate data is available. Which method is better depends on the line of insurance and the situation. 3. True.

4. Prior to inflation:

Loss	Probability	Contribution to Layer 0-100	Contribution to Layer 100-250	Contribution to Layer 250-∞
50	0.6	50	0	0
100	0.3	100	0	0
250	0.1	100	150	0
Weighted Average		70	15	0

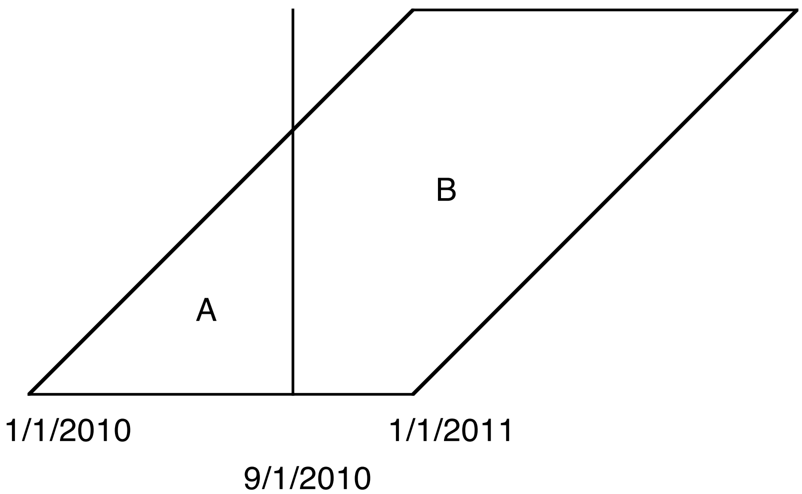
For the next year, increase each size of loss by 8%:

Loss	Probability	Contribution to Layer 0-100	Contribution to Layer 100-250	Contribution to Layer 250-∞
54	0.6	54	0	0
108	0.3	100	8	0
270	0.1	100	150	20
Weighted Average		72.4	17.4	2

Trend for layer from 0 to 100 is: 72.4/70 - 1 = **3.4%.**
Trend for layer from 100 to 250 is: 17.4/15 - 1 = **16.0%.**
Trend for layer from 250 to ∞ is: 2/0 - 1 = **∞.**
Comment: Similar to 6, 5/99, Q.39.

5. 1. Have conditions changed since the last occurrence?
2. Has the coverage changed so that the same event would produce different losses?
3. Have the exposures changed?
4. Is the frequency of the event more credible, so that a separate estimate can be made of the future severity, while using the past purely as a basis to derive a frequency estimate?
Or vice versa?
5. Are there simple adjustments that can be made to past experience data?
Or is the peril so volatile that a more fundamental method of loss estimation would be superior to simply assuming that the loss experience of the past 30 years or so would repeat itself?
6. Is there a base coverage that can be used so that catastrophe losses can be estimated as a percentage of that base?
Or is the peril so unusual that it is better to treat it totally separately, and add a measure of that peril's expected losses to the regularly measured expected losses?
7. Credibility of an individual insurer's own experience versus that of the insurance industry.
8. Should one use a composite rate or separate rate for the catastrophe peril.
Comment: Give only four issues. Based on "Catastrophe Ratemaking," by Michael Walters, formerly on the syllabus.

6. a. Under the current law, a worker with 80% of the SAWW gets the minimum benefit.
60%/75% = 80%.
A worker with 1.2 times the SAWW gets the maximum benefit. 90%/75% = 120%.
Thus 40.05% of workers get the minimum, and 1 - 71.14% = 28.86% get the maximum.
The workers who get in between the minimum and the maximum have a portion of wages of:
53.76% - 23.13% = 30.63%.
Average current benefit is SAWW times:
(40.05%)(60%) + (28.86%)(90%) + (75%)(30.63%) = **72.98%**.
b. Under the proposed law, a worker with half the SAWW gets the minimum benefit, and a worker with 2 times the SAWW gets the maximum benefit.
11.33% of workers would get the minimum, and 1 - 96.91% = 3.09% would get the maximum.
The workers who would get in between the minimum and the maximum have a portion of wages of:
92.48% - 4.00% = 88.48%.
Average proposed benefit is SAWW times:
(11.33%)(30%) + (3.09%)(120%) + (60%)(88.48%) = 60.20%.
Direct effect is: 60.20/72.98 - 1 = **17.5% decrease**.
c. 8/12 of the accidents occur prior to September 1, while 4/12 occur after.
Thus the average benefit level for AY2010 is: (2/3)(1) + (1/3)(0.825) = 0.942.
The adjustment factor to bring AY2010 losses to the current benefit level is: 0.825/0.942 = **0.876**.
For Policy Year 2010, we draw a diagram, similar to those used to calculate the effect on premiums of law amendments that effect all outstanding policies. Here the benefit change effects all accidents after a certain date.



Area A is: (1/2)(2/3)(2/3) = 2/9. Area B = 1 - 2/9 = 7/9.
Thus the average benefit level for PY2010 is: (2/9)(1) + (7/9)(0.825) = 0.864.
The adjustment factor to bring PY2010 losses to the current benefit level is: 0.825/0.864 = **0.955**.

d. $\frac{\text{percentage of wages}}{\text{percentage of workers}} = \frac{92.48\% - 38.80\%}{96.61\% - 57.47\%} = \mathbf{137.1\%}$.

Alternately, assume 10,000 workers with a state average weekly wage of $1000,
for a total of $10 million in weekly wages.
Then, the number of workers making between the SAWW and twice the SAWW is:
(96.61% - 57.47%)(10,000) = 3914.
The weekly wages of workers making between the SAWW and twice the SAWW is:
(92.48% - 38.80%)($10 million) = $5,368,000.
Average weekly wage for these workers is: $5,368,000/3914 = $1371.
As a percent of the SAWW this is: $1371/$1000 = **137.1%**.
Comment: Information was taken from the 1998 Massachusetts Wage Distribution Table.
Part d is not something I would expect you to be asked on this exam; it is analogous to the calculation of the average size of claims for those claims in a size interval, as per Loss Distributions on Exam 4.

7. The loss development method is based upon the assumption that claims move from unreported to reported-and-unpaid to paid in a pattern that is sufficiently consistent that past experience can be used to predict future development. It is also assumed that the rate at which claims are paid and the adequacy of case reserves is sufficiently consistent. In other words, we assume that the present paid plus case reserves have the approximately the same degree of reserve adequacy as did the past paid plus case reserves losses on which the development factors are based.

8. First adjust all of the losses to the 4/1/11 level.

Accident Year	Reported Losses	Years of Trend	Trended Losses
2007	$20 million	3.75	25.776 million
2008	$25 million	2.75	30.112 million
2009	$30 million	1.75	33.771 million

Accident Date	Reported Losses	Trend Period	Trended Losses
2/1/07	$1,000,000	4 years and 2 months	1.326 million
10/15/07	$450,000	3 years and 5.5 months	0.569 million
4/1/09	$700,000	2 years	0.801 million
8/15/09	$500,000	1 year and 7.5 months	0.558 million

Excess Losses for AY07 are: (1.326 - 0.5) + (0.569 - 0.5) = 0.895.

Then, the Excess Ratio for AY07 is: 0.895/24.881 = 3.60%.

Accident Year	Trended Losses	Excess Losses	Non-Excess Losses	Excess Ratio
2007	25.776	0.895	24.881	3.60%
2008	30.112	0	30.112	0.00%
2009	33.771	0.359	33.412	1.07%
Average				**1.56%**

Comment: See Exhibit 6.3 in Basic Ratemaking, which ignores severity trends.

Many more years would normally be used.

2009 losses are at first report. For many lines of insurance, the excess ratio increases on average at later reports. Therefore, it might be better to exclude first report from such a calculation.

One would also have to take care that the loss development factors would be appropriate to develop immature non-excess losses to ultimate non-excess losses. Thus one would have to remove excess losses from the loss development triangle.

9. AY 2002 is currently at 48 months (the time from 1/1/02 to 12/31/05).

AY 2002 has a current case incurred of: $12,000,000 + $2,000,000 = $14 million.

AY 2002 has a projected ultimate of $15 million.

⇒ the loss development factor from 48 months to ultimate is: 15/14 = 1.071.

AY 2003 is currently at 36 months, with a case incurred of: $7 million + $4 million = $11 million.

Projected ultimate accident year 2003 losses are: (1.06)(1.071)($11 million) = **$12.49 million**.

AY 2004 is currently at 24 months, with a case incurred of: $5 million + $5 million = $10 million.

Projected ultimate accident year 2004 losses are:

(1.18)(1.06)(1.071)($10 million) = **$13.40 million**.

AY 2005 is currently at 12 months, with a case incurred of: $2 million + $6 million = $8 million.

Projected ultimate accident year 2005 losses are:

(1.53)(1.18)(1.06)(1.071)($8 million) = **$16.40 million**.

Comment: Similar to 5, 5/03, Q.12.

10. a. Since $X < L$, prior to inflation this loss contributes 0 to this layer. Since $X(1 + T) > L$, after inflation the loss exceeds the lower end of the layer. After inflation, this loss contributes:
$(1 + T)X - L > 0$ to this layer. The rate of increase in the layer is **infinite**.
b. Since $X(1 + T) < U$, after inflation the loss still does not exceed the upper end of the layer. Prior to inflation this loss contributes $X - L$ to this layer. After inflation, this loss contributes $(1 + T)X - L$ to this layer. The increase in the contribution is TX. The rate of increase in the layer is: **$TX/(X - L)$**.
c. Since $X(1 + T) > U$, after inflation this loss would contribute its width to the layer: $U - L$.
Rate of increase of the contribution to the layer is: $\{(U - L) - (X - L)\}/(X-L)$ = **$(U - X)/(X - L)$**.
<u>Comment</u>: For example, let L = 5000, U = 20,000, and T = 25%.
Then in part b, the loss before inflation is of size between 5000 and 20,000/1.25 = 16,000.
After inflation the loss is of size between (5000)(1.25) = 6250 and (16,000)(1.25) = 20,000.
The contribution to the layer was X - 5000 before inflation, and is 1.25X - 5000 after inflation.
The increase of the contribution is: 0.25X. The rate of increase in the layer is: 0.25X/(X - 5000).

11. (a) The incurred losses for calendar year 5 are the losses paid in year 5 plus the change in reserves during year 5.
This is AY5 + change in AY4 + change in AY3 + change in AY2 + change in AY1:
126.08 + (218.33 - 164.67) + (283.70 - 253.04) + (573.22 - 537.09) + (737.27 - 727.62) = **256.18 million**.
(b) The incurred losses for calendar year 6 are the losses paid in year 6 plus the change in reserves during year 6.
This is AY6 + change in AY5 + change in AY4 + change in AY3 + change in AY2:
138.72 + (177.31 - 126.08) + (250.50 - 218.33) + (307.45 - 283.70) + (587.68 - 573.22) = **260.33 million**.
(c) For example, 593.30/377.47 = 1.5718.

AY	12	24	36	48	60	Ultimate
1	377.47	593.30	686.29	727.62	737.27	**737.27**
2	342.26	480.02	537.09	573.22	587.68	**587.68**
3	194.34	253.04	283.70	307.45		**313.37**
4	164.67	218.33	250.50			**273.30**
5	126.08	177.31				**219.76**
6	138.72					**241.00**
1	1.5718	1.1567	1.0602	1.0133		
2	1.4025	1.1189	1.0673	1.0252		
3	1.3020	1.1212	1.0837			
4	1.3259	1.1473				
5	1.4063					
Average	1.4017	1.1360	1.0704	1.0192	1.0000	
Factor to Ultimate	1.7373	1.2394	1.0910	1.0192	1.0000	

For example, the 36 month to ultimate factor is: (1.0704)(1.0192) = 1.0910.
For example, the AY6 estimated ultimate is: (1.7373)(138.72) = $241.00 million.
(d) We would expect the case reserves reported during Year 6 to be somewhat more adequate than average. Therefore, one would expect somewhat less development than the historical average for AY6 from 12 months to ultimate. The same phenomenon would apply to a lesser extent to AY5 from 24 months to ultimate, etc. In addition, age-age development factors along the latest diagonal are probably somewhat higher than expected due to reserve strengthening during the hard market. For all these reasons, one should decrease somewhat the estimates of ultimate for immature years. (For example, one might select age to age factor of: 1.36, 1.12, 1.05, and 1.01. This would result in estimated ultimates of: AY3 310.52,
AY4 265.66, AY5 210.60, AY6 224.08.)

(e) For example, 212404/188352 = 1.1277.

AY	12	24	36	48	60	Ultimate
1	188,352	212,404	225,461	229,128	229,139	**229,139**
2	165,808	197,177	207,916	211,680	211,701	**211,701**
3	91,949	113,081	118,538	121,171		**121,180**
4	74,850	90,194	95,870			**97,685**
5	52,333	62,731				**67,548**
6	54,984					**70,460**
1	1.1277	1.0615	1.0163	1.0000		
2	1.1892	1.0545	1.0181	1.0001		
3	1.2298	1.0483	1.0222			
4	1.2050	1.0629				
5	1.1987					
Average	1.1901	1.0568	1.0189	1.0001	1.0000	
Factor to Ultimate	1.2815	1.0768	1.0189	1.0001	1.0000	

For example, the 24 month to ultimate factor is: (1.0568)(1.0189)(1.0001) ≅ 1.0768.

For example, the AY6 estimated ultimate is: (1.2815)(54,984) ≅ 70,460.

(f) $\hat{\beta} = \{N\Sigma X_i Y_i - \Sigma X_i \Sigma Y_i\} / \{N\Sigma X_i^2 - (\Sigma X_i)^2\} = \{(6)(64,508) - (21)(18051)\}/\{(6)(91) - 21^2\} = 75.97$.

$\hat{\alpha} = \overline{Y} - \hat{\beta}\,\overline{X} = (18501/6) - (75.97)(3.5) = 2742.6$.

AY	Ultimate Loss & ALAE	Ultimate Claims	Average Size	Square of Year	Avg. Size times Year	Fitted Severity
1	737.27	229,139	3217.6	1	3,218	2818.6
2	587.68	211,701	2776.0	4	5,552	2894.5
3	313.37	121,180	2586.0	9	7,758	2970.5
4	273.30	97,685	2797.8	16	11,191	3046.5
5	219.76	67,548	3253.4	25	16,267	3122.4
6	241.00	70,460	3420.4	36	20,522	3198.4
Sum			18,051	91	64,508	

Annual Severity Trend Factor (AY6/AY5 Least-Squares) = 3198.4/3122.4 = **1.0243**.

Comment: There are various ways to select development factors, other than the one used here. You are unlikely to be asked to do a regression on this exam. For part d, see page 4 of "A Macroeconomic View of the Insurance Marketplace," by Boor. It would be useful to have separate data on paid losses and case reserves. In general, having more information about this insurer, state, line of business, etc., should allow an actuary to improve his estimates of ultimate losses.

If one fits a straight line, as opposed to an exponential curve, then it would make more sense to me to project the fitted value forward to the proposed effective period.

Assume for example that the average date of loss for policies to be written under the new rates were time 8. Then the fitted severity for time 8 is: 2742.6 + (8)(75.97) = 3350.4.

Then a severity trend factor to be applied to AY5 loss plus alae would be: 3350.4/3122.4 = 1.0730. This differs from 1.0243^3 = 1.0747, doing things as per my solution.

12. Some disadvantages of using actual experience:
1. Large process variance from only a few actual occurrences.
2. Coverages may have changed from decades ago.
Some advantages of using computer modeling:
1. Can simulate thousands of years of data.
2. Can be used to get fine detail by location (for example zip code) in order to help construct territories and to analyze territory rates.
3. The model can be run on a base class house, which simplifies the analysis.
Comment: Based on "Catastrophe Ratemaking," by Michael Walters, formerly on the syllabus.

13. **$12,000** from Policy C.

14. $3000 from Policy A, $10,000 from B, and $5000 from C, for a total of **$18,000**.

15. $3000 from Claim #1, and $1000 from #3, for a total of **$4,000**.

16. $3000 - $5000 = - $2000 from Claim #1, $9000 - $5000 = $4000 from #2, $1000 from #3, and $10,000 from #4, for a total of **$13,000**.

17. **$1000** from Claim #3.

18. $1000 from Claim #3, and $15,000 from #4, for a total of **$16,000**.

19. $1000 from Claim #3, and $10,000 from #4, for a total of **$11,000**.

20. $1000 from Claim #3, and $15,000 from #4, for a total of **$16,000**.
Comment: At ultimate the Accident Year paid and incurred losses are always equal.

21. **$12,000** from Policy C.
Comment: If these policies had been subject to premium audits, then the Policy Year 2006 written premium as of 12/31/06 would have differed from its value at ultimate.

22. **$5,000** from Policy C.

23. **$12,000** from Policy C.
Comment: At ultimate the Policy Year written and earned premiums are always equal.

24. **$0**.

25. $15,000 from Claim #4, and $3000 from #5, for a total of **$18,000**.

26. **$10,000** from Claim #4.

27. $15,000 from Claim #4, and $3000 from #5, for a total of **$18,000**.
Comment: At ultimate the Policy Year paid and incurred losses are always equal.

28. **$1000** from Claim #3.

29. $1000 from Claim #3, and $15,000 from #4, for a total of **$16,000**.

30. $1000 from Claim #3, and $10,000 from #4, for a total of **$11,000**.

31. $1000 from Claim #3, and $15,000 from #4, for a total of **$16,000**.
Comment: At ultimate the Report Year paid and incurred losses are always equal.

32. For convenience, let us assume a total of 10,000 policies.
Then there are 9000 renewal policies, providing (initially) coverage to (9000)(1.8) = 16200 cars, starting on January 1.
By the end of the year (24%)(16200) = 3888 cars have cancelled.
The exposures at time t, $0 \le t \le 1$ is: 16200 - 3888t.
The total earned exposures are: $\int_0^1 16200 - 3888t \, dt = 16200 - 3888/2 = 14256$ car years.

The average time of earning these exposures is:

$$\int_0^1 t(16200 - 3888t) \, dt \; / \; 14256 = (16200/2 - 3888/3)/14256 = 0.4773.$$

There are 1000 new policies, providing (initially) coverage to (1000)(1.2) = 1200 cars, starting at time x, with x uniformly distributed on (0, 1).
For a new policy written at time x, the probability of still being in force at time t, $1 > t > x$ is:
1 - (.24)(t - x). The total earned exposures for new policies is:

$$1200 \int_{x=0}^{1} \int_{t=x}^{1} 1 - .24(t - x) \, dt \, dx = 1200 \int_{x=0}^{1} (1 - x) - .12(1 - x)^2 \, dx = (1200)(.46) = 552.$$

The average time of earning these exposures is:

$$\int_{x=0}^{1} \int_{t=x}^{1} t\{1 - .24(t - x)\} \, dt \, dx \; / \int_{x=0}^{1} \int_{t=x}^{1} 1 - .24(t - x) \, dt \, dx =$$

$$\int_{x=0}^{1} .42 + .12x - .5x^2 - .04x^3 \, dx \; / \; 0.46 = 0.30333/0.46 = 0.6594.$$

The average date of earning is: {(14,256)(0.4773) + (552)(0.6594)} / (14,256 + 552) = 0.4841.
Thus one should **trend losses 0.4841 years beyond the beginning of 2007**.
This is about (.5 - .4841)(12) = .191 months = (.191)(30) = 6 days before the middle of the year, or a trend to date for losses of about **June 24 or June 25**, 2007.
Comment: Similar to 5, 5/05, Q.16. However, its many complications make my question well beyond what you are likely to be asked on your exam.

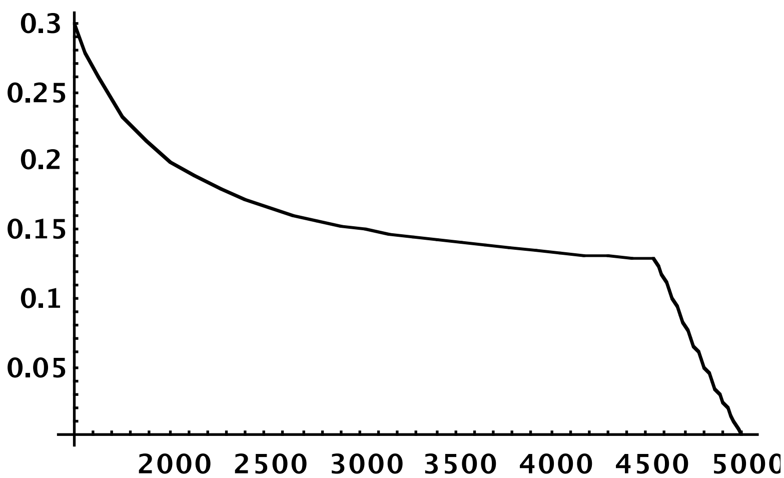
33. a. (60 million)(1.1) = **$66 million**.
b. Assume that the basic limit losses for those who purchased 20/40/10 limits were 30% of the total basic limits losses. (30%)(60) = $18 million.
Assume that these insureds losses will be 1.1 times as much when they purchase 25/50.
($18 million)(1.1) = $19.8 million. $80 million + $19.8 million - $18 million = **$81.8 million**.
c. Insurance will become more expensive for those currently buying basic limits.
This may lead some of them to become uninsured.
Therefore, the rates for uninsured motorists coverage may increase.
Alternately, the increase in financial responsibility limit may lead some insureds to reexamine the limits they currently purchase. Some of these will decide to increase their limits. This will lead to a change in the mix of people buying the various higher limits, which may in turn change the appropriate increased limit factors.
Comment: I have assumed that the increased limit factor is proportional to the increase in losses.
The insureds who currently bought only 20/40 limits were self-selected. They have different characteristics than those who bought 25/50 limits. Therefore, their ratio of B.I. losses at the new limits to those at the old limits may differ from 1.1.
Any of the assumptions I made in part b can be questioned. In part c, one could point out that the losses under underinsured motorist coverage will probably go down, since everyone will now have at least 25/50 B.I. limits. However, I am not sure that this is an indirect effect.

34. The average trended pure premium is **$17.75**.

Calendar Year	Car Years (000)	Paid Hail Losses ($ million)	Trend Period	Trended Losses	Pure Premium
1995	100	1.2	16	2.2476	$22.48
1996	110	0	15	0.0000	$0.00
1997	120	0	14	0.0000	$0.00
1998	130	0	13	0.0000	$0.00
1999	140	1.5	12	2.4015	$17.15
2000	150	0	11	0.0000	$0.00
2001	160	0.1	10	0.1480	$0.93
2002	170	4.3	9	6.1202	$36.00
2003	180	1.9	8	2.6003	$14.45
2004	190	0	7	0.0000	$0.00
2005	200	9.1	6	11.5144	$57.57
2006	210	0	5	0.0000	$0.00
2007	220	0	4	0.0000	$0.00
2008	230	3.5	3	3.9370	$17.12
2009	240	0.6	2	0.6490	$2.70
Average					**$11.23**

Comment: I have ignored any changing impact of deductibles.
Hail losses would be removed from the most recent year of comprehensive losses prior to using them to predict the future non-hail losses. The expected loss pure premium due to hail losses would be added to that estimated for non-hail losses.

35. Let X be the size of loss prior to inflation.
If X < 1000/1.1 = 909, then both prior to inflation and after inflation, the contribution to this layer is 0.
The rate of increase of the contribution is either undefined or zero.
If 909 < X < 1000, then prior to inflation the contribution to this layer is 0, but after inflation, the contribution to this layer is: 1.1X - 1000 > 0. The rate of increase of the contribution is undefined.
If 1000 < X < 5000/1.1 = 4545, then prior to inflation the contribution to this layer is: X - 1000, and after inflation, the contribution to this layer is: 1.1X - 1000.
The rate of increase of the contribution is: .1X/(X - 1000).
If 4545 < X < 5000, then prior to inflation the contribution to this layer is: X - 1000, and after inflation the contribution to this layer is: 5000 - 1000 = 4000.
The rate of increase of the contribution is: (5000 - X)/(X - 1000).
If X > 5000, then both prior to inflation and after inflation, the contribution to this layer is 4000.
The rate of increase of the contribution is: 0.
Comment: .1X/(X - 1000) > .1, the overall rate of inflation; as X → 1000, this rate of increase → ∞.
(5000 - X)/(X - 1000), for 4545 < X < 4636, is greater than .1, the overall rate of inflation.
(5000 - X)/(X - 1000), for 4636 < X < 5000, is less than .1, the overall rate of inflation.
Here is a graph of the rate of increase as a function of X, for 1500 < X < 5000:



36. "For liability coverage, an example would be a very large total limits loss. The remedy is usually to exclude the high severity amount, and only retain the basic limits portion of the loss in the regular ratemaking exercise.
For an extraordinary type of liability loss, such as environmental or asbestos, the method would be to eliminate all such losses, and separately measure the total expected loss from that cause of loss in the future.
For unusual property perils within a broad property line of business, such as hurricane, tornado, winter freeze, or sinkhole, the actual losses need to be removed from the relatively short experience review period, so as to estimate the future expected losses without the short-term bias of the recent past," (and then load in a long term expected value.)
Comment: Give only one example for liability and one for property.
Based on "Catastrophe Ratemaking," by Michael Walters, formerly on the syllabus.

37. I used the average of all available link ratios.
The estimated ultimate losses for accident year 2009 are: (1.0645)(2000) = **$2129**.

Accident Year	Age 12	Age 24	Age 36	Age 48	Age 60
1995	$1000	$1100	$1070	$1070	$1070
1996	$1250	$1360	$1310	$1310	
1997	$1600	$1750	$1700		
1998	$1700	$1890			
1999	$2000				
Accident		Link Ratios			
Year	12 to 24	24 to 36	36 to 48	48 to 60	
1995	1.1000	0.9727	1.0000	1.0000	
1996	1.0880	0.9632	1.0000		
1997	1.0938	0.9714			
1998	1.1118				
Avg.	1.0984	0.9691	1.0000	1.0000	
Ultimate Factor	1.0645	0.9691	1.0000	1.0000	
Accident Year	Loss & ALAE @ 12/31/99	Ultimate Factor	Projected Ultimate		
1995	$1070	1.0000	$1070		
1996	$1310	1.0000	$1310		
1997	$1700	1.0000	$1700		
1998	$1890	0.9691	$1832		
1999	$2000	1.0645	**$2129**		

Comment: Other selected factors would be reasonable.
The paid losses decline at 36 months presumably due to salvage and subrogation.

38. (1.1)(127,470) = 140,217. 13,484/140,217 = 9.62%.

Accident Year	Reported Losses	Devel. Factor	Ultimate Losses	Reported ALAE	Devel. Factor	Ultimate ALAE	ALAE Ratio
2007	127,470	1.10	140,217	12,039	1.12	13,484	9.62%
2008	148,466	1.22	181,128.5	13,743	1.25	17,179	9.48%
2009	151,700	1.50	227,550	14,286	1.55	22,143	9.73%

A reasonable provision would be the average: (9.62% + 9.48% + 9.73%)/3 = **9.6%**.

39. B. While frequency and severity trends are often analyzed separately, it is sometimes preferable to look at trends in the pure premium, thus combining the impacts of frequency and severity

40. Multiply each loss by 1.20.

Loss	Contribution to Layer 100 xs 250		Inflated Loss	Contribution to Layer 100 xs 250
75,000	0		90,000	0
180,000	0		216,000	0
220,000	0		264,000	14,000
270,000	20,000		324,000	74,000
310,000	60,000		372,000	100,000
500,000	100,000		600,000	100,000
800,000	100,000		960,000	100,000
Sum	280,000		Sum	388,000

The annual claims inflation rate in the layer $100,000 excess of $250,000 is:
388/280 - 1 = **38.6%**.
Comment: Similar to 5, 5/04, Q.45.

41. Averaging all available link ratios, the age-to-ultimate development factor for accident year 2005 as of December 31, 2005 is: (1.5533)(1.0619)(1.0402) = **1.7158**.

Accident Year	12	24	36	48
2001	$112	$181	$193	$199
2002	$124	$192	$203	$213
2003	$138	$209	$222	
2004	$131	$201		
2005	$142			
2001	1.6161	1.0663	1.0311	
2002	1.5484	1.0573	1.0493	
2003	1.5145	1.0622		
2004	1.5344			
Average	1.5533	1.0619	1.0402	
Factor to Ultimate	**1.7158**	1.1046	1.0402	

The ultimate loss amount for accident year 2005 is: (1.7158)($142) = $243.64.
The average date of writing is 1/1/07. The average date of loss is 7/1/07.
The trend period is 2 years, from 7/1/05 to 7/1/07.
The trended ultimate loss amount for accident year 2005 is:
$(1.03^2)(1.02^2)$($243.64) = **$268.9**.
Comment: Similar to 5, 5/04, Q.37. Somewhat different selections could have been made than taking the average of all available link ratios, as I did.

42. (a) Starting with Accident Year 2008, paid medical losses will develop less.
In order to quantify how much less future development will be compared to historical development one would attempt to estimate:
What percent of historical medical losses are due to chiropractic visits, physical therapy visits, and occupational therapy visits? (Either in combination or separately.)
For these types of visits, what percent of historical losses are due to visits beyond the 24th? (Preferably separately by chiropractic, physical therapy, and occupational therapy.)
In what percent of cases that would exceed 24 visits will exceptions be allowed by the insurer?
For those claims which exceeded 24 visits, how long on average did it take to exceed the 24th visit? (In order to estimate the impact on second to ultimate factors, 3rd to ultimate factors, etc.)
(b) Starting with Accident Year 2008, paid indemnity losses will develop more due to the increase in the temporary partial limit and will develop less due to the decrease in the temporary total limit and the combined limit.
What will happen in total will depend on the mix of dollars by injury type.
In order to quantify how much less future development will be compared to historical development one would attempt to estimate:
What percent of historical indemnity losses are due to temporary partial disability?
What percent of historical indemnity losses are due to temporary total disability?
What percent of historical indemnity losses are due to workers who received temporary total disability but not temporary partial disability?
What percent of historical indemnity losses are due to workers who received temporary partial disability but not temporary total disability?
What percent of historical indemnity losses are due to workers who received both temporary partial disability and temporary total disability?
What percent of historical indemnity losses for temporary partial disability are due to the listed exceptions?
What percent of historical indemnity losses for temporary total disability are due to the listed exceptions?
What is the current payment pattern from date of injury for temporary partial disability?
What is the current payment pattern from date of injury for temporary total disability?
What is the current payment pattern from date of injury for workers who receive at some time both temporary total and partial disability payments?
Comment: Somewhat similar to 5, 5/07, Q.22, but beyond what you are likely to be asked on your exam. There are somewhat different full-credit answers one could give for the type of information to be collected. *In practical applications, this type of impact can be a difficult thing to estimate. In practical applications, there are limits on how much information one can collect and its reliability. One important source of information is to talk to claims handlers. There are many possible complications. For example, with a shorter limit on temporary total disability payments, could there be an increase in the number of applications for permanent total disability?*

43. Average the last three loss development factors as a balance between stability and responsiveness.

	15	27	39	51
Link Ratio	1.193	1.081	1.027	1.004
Factor to ultimate	1.330	1.115	1.031	1.004

The average date of accident under the new rates is September 1, 2006.

Apply the trend as per an exponential trend. $1.04^{4.167} = 1.178$.

Year	Loss & ALAE	Devel. Factor	Trend Period	Trend Factor	Trended & Developed	ULAE Factor	Loss & LAE
2002	$3052	1.031	4.167	1.178	$3705	1.09	$4039
2003	$4260	1.115	3.167	1.132	$5378	1.09	$5862
2004	$3610	1.330	2.167	1.089	$5227	1.09	$5698

Year	Loss & LAE	Premium	Loss & LAE Ratio
2002	$4039	$5525	**73.1%**
2003	$5862	$7252	**80.8%**
2004	$5698	$6765	**84.2%**

Comment: Similar to 6, 5/94, Q.36. There is not one single correct selection.

44. For the losses limited to $100,000:

Original Loss	Prob.	Limited Losses	Inflated Loss	Limited Losses
10,000	65%	10,000	11,000	11,000
50,000	30%	50,000	55,000	55,000
250,000	5%	100,000	275,000	100,000
Average		26,500		28,650

The limited losses increase: 28,650/26,500 - 1 = 8.1%.

The layer $100,000 excess of $100,000 is from $100,000 to $200,000.

Original Loss	Prob.	Contribution to Layer	Inflated Loss	Contribution to Layer
10,000	65%	0	11,000	0
50,000	30%	0	55,000	0
250,000	5%	100,000	275,000	100,000
Average		5,000		5,000

The losses in this layer do not increase.

The limited losses increase faster than the losses in the layer $100,000 excess of $100,000.

Comment: Similar to 5, 5/06, Q.31.

The losses excess of $100,000 would increase faster than the overall rate of inflation of 10%.

Original Loss	Prob.	Excess Losses	Inflated Loss	Excess Losses
10,000	65%	0	11,000	0
50,000	30%	0	55,000	0
250,000	5%	150,000	275,000	175,000
Average		7,500		8,750

The excess losses increase: 8750/7500 - 1 = 12.5%.

Basic limit losses increase slower than the overall rate of inflation.

Excess limits losses increase faster than the overall rate of inflation.

However, the behavior of a layer with a bottom other than zero and a top other than infinity, such as 100,000 excess of 100,000, depends on the particular size of loss distribution.

45. E. The selected development factors are assumed to be those that will produce the best estimates of ultimate losses when applied to past Accident Years. Now we need to select the trend factor that is the best estimate of the ratio of the level of ultimate losses during the time the rates will be in effect to the ultimate losses for past Accident Year(s). There is no overlap.

46. E. Average date of writing: 12/1/93. Average date of loss: 3/1/94 (6 month policies).

Trend period: 7/1/92 to 3/1/94: 20 months.

47. (a) i. Advantages: Calendar Year is already collected for financial reporting purposes. Calendar Year data is available quickly and is not subject to loss development.
Disadvantages: However, Calendar Year losses consists of data from policies written at many different times, which for other than short-tailed lines of insurance are not a very good match for the policies that contribute to CY premiums.
ii. Advantages: Calendar/Accident Year data consists of losses from a given set of accidents. Calendar/Accident Year data is available more quickly than Policy Year data. Calendar Year premiums are an approximate match to Accident Year losses.
Disadvantages: Calendar/Accident Year data is available more slowly than Calendar Year data. One has to develop Accident Year losses to ultimate. Calendar Year premiums are an approximate match to Accident Year losses, but not as good a match as PY losses to PY premiums, particularly for long-tailed lines of insurance.
iii. Advantages: Policy Year data matches premiums from a set of policies with losses from the same set of policies.
Disadvantages: Policy Year data is not as available as quickly as Calendar/Accident Year data. One has to develop Policy Year losses to ultimate.
iv. Advantages: For Report Year data there are no unknown claims by the end of the Report Year, and thus no pure IBNR; therefore, there is less loss development to ultimate.
Disadvantages: There is no good match for Report Year losses on the premium side.
A Report Year consists of contributions from many different AYs and many different PYs.
(b) Collision: Calendar Year. Auto B.I. Liability: Calendar/Accident Year.
General Liability: Policy Year. Medical Malpractice: Report Year.
Comment: Collision claims are reported and settled quickly, and thus Calendar Year data is often used. Other lines for which Calendar Year data are commonly used include: Health, Other Than Collision (Comprehensive), and Fire. Most actuaries use Accident Year data for Auto B.I. Liability. Most actuaries use Policy Year data for General Liability.
Many actuaries use use Report Year data for Medical Malpractice Insurance. As lines of insurance get longer tailed, actuaries doing ratemaking tend to move from CY to AY to PY to RY data.

48. Assume 63 months is ultimate. Average the last three loss development factors as a balance between stability and responsiveness.

	15	27	39	51
Link Ratio	1.153	1.078	1.034	1.009
Factor to ultimate	1.297	1.125	1.043	1.009

The average date of accident under the new rates is July 1, 1995.

Apply the trend as per an exponential trend. $1.07^4 = 1.311$.

Year	Loss & ALAE	Devel. Factor	Trend Period	Trend Factor	Trended & Developed	ULAE Factor	Loss & LAE
1991	$2052	1.043	4	1.311	$2805	1.11	$3114
1992	$2260	1.125	3	1.225	$3115	1.11	$3457
1993	$2610	1.297	2	1.145	$3876	1.11	$4302

Year	Loss & LAE	Premium	Loss & LAE Ratio
1991	$3114	$3525	**88.3%**
1992	$3457	$4252	**81.3%**
1993	$4302	$5765	**74.6%**

Alternately, assume the 63 months to ultimate factor is the same as the 51 to 63 factor.
Average the last two loss development factors in order to be responsive.

	15	27	39	51	63
Link Ratio	1.154	1.078	1.030	1.015	1.015
Factor to ultimate	1.320	1.144	1.061	1.030	1.015

Apply the trend as per a linear trend. $1 + (4)(.07) = 1.28$.

Year	Loss & ALAE	Devel. Factor	Trend Period	Trend Factor	Trended & Developed	ULAE Factor	Loss & LAE
1991	$2052	1.061	4	1.280	$2787	1.11	$3093
1992	$2260	1.144	3	1.210	$3128	1.11	$3473
1993	$2610	1.320	2	1.140	$3928	1.11	$4360

Year	Loss & LAE	Premium	Loss & LAE Ratio
1991	$3093	$3525	**87.8%**
1992	$3473	$4252	**81.7%**
1993	$4360	$5765	**75.6%**

Comment: There is not one single correct selection. I have shown two reasonable choices.

49. 7/1/95 is two years past the average accident date for AY93 of 7/1/93.
The projected frequency is: .136 + (2)(.004) = .144.
The frequency trend for two years is: .144 /.136 = 1.059.
The annual severity trend is: $(1377/1064)^{1/5}$ = 1.053.
The severity trend for two years is: 1.053^2 = 1.109.
The trend factor for two years is: (1.059)(1.109) = **1.174**.
Alternately, the annual frequency trend is: 0.136/0.132 = 1.030.
The trend factor for two years is: $\{(1.030)(1.053)\}^2$ = **1.176**.

50. C. The incurred losses for calendar year 1992 are the losses paid in 1992 plus the change in reserves during 1992.
This is AY92 + change in AY91 + change in AY90 + change in AY89 + change in AY88:
2750 + (2950-2500) + (3000-2500) + (1775-1700) + (1810-1800) = **3785**.
Comment: Basic idea you have to know! Accident Years before 1988 do not contribute to CY92 since 60 months is given as ultimate.

51. 1. 7/1/94 to 1/1/97: Average experience period accident date to average effective period accident date. Inflation is measured by the trend factor.
2. 1/1/97 to 10/1/97: First nine months after effective period accident date. Inflation during this period is measured by the change in estimates during the comparable period between the average experience accident date and the experience period evaluation date:
7/1/94 to 3/31/95.
3. 10/1/97 to Ultimate: Inflation during this period is measured with the loss development factor.
Since there is no overlap in time periods, there is no overlap in the recognition of inflation in ratemaking.
Comment: Intended to be answered out of Cook "Trend and Loss Development Factors," no longer on the syllabus; however, it should still be able to be answered out of the somewhat briefer discussion of the overlap fallacy currently on the syllabus.

52. Assume for simplicity that the expected frequency is 5. (Since we will be taking the ratio of two years, this assumption does not affect the answer.)
Then we expect one loss of each size.

Loss	Contribution to Layer 0-50	Contribution to Layer 50-100	Contribution to Layer 100-200	Contribution to Layer 200-∞	Total
40	40	0	0	0	40
80	50	30	0	0	80
120	50	50	20	0	120
160	50	50	60	0	160
200	50	50	100	0	200
Total	240	180	180	0	600

For the next year, increase each size of loss by 10%:

Loss	Contribution to Layer 0-50	Contribution to Layer 50-100	Contribution to Layer 100-200	Contribution to Layer 200-∞	Total
44	44	0	0	0	44
88	50	38	0	0	88
132	50	50	32	0	132
176	50	50	76	0	176
220	50	50	100	20	220
Total	244	188	208	20	660

Trend for layer from 0 to 50 is: 244/240 - 1 = **1.7%.**
Trend for layer from 50 to 100 is: 188/180 - 1 = **4.4%.**
Trend for layer from 100 to 200 is: 208/180 - 1 = **15.6%.**
Comment: Could be answered on an earlier exam. Note that the limited losses in the layer from 0 to 50 increase slower than the overall rate of inflation 10%. The excess losses in the layer from 200 to ∞ increase faster than the overall rate of inflation. The losses in middle layers, such as 50 to 100 and 100 to 200, can increase either slower or faster than the overall rate of inflation, depending on the particulars of the situation.

53. D. The average date of accident for AY 1999 is 7/1/99.
The new rates will be in effect from 7/1/01 to 4/1/02, with average date of writing 11/15/01.
The average accident date is 18/2 = 9 months later, 8/15/02.
From 7/1/99 to 8/15/02 is 3 years and 1.5 months, or **37.5 months**.
Comment: Unless specifically stated otherwise, as was done here, assume the new rates will be in effect for one year.

54. a. If the policies are 6-month, the average accident date is: 11/15/01 plus 3 months = 2/15/02.
Trend period is: from 7/1/99 to 2/15/02 = **31.5 months**.
b. If the policies are 12-month, the average accident date is: 11/15/01 plus 6 months = 5/15/02.
Trend period is: from 7/1/99 to 5/15/02 = **34.5 months**.

55. a) 1. Policy year compiles premiums written in a year and the losses for those policies regardless of when they are reported.
2. Calendar year – Earned premium is the premium written during the year plus the unearned premium reserve at beginning of this year minus unearned premium reserve reserve at end of the year. The incurred losses are the paid losses during the year plus the loss reserve at the end of the year minus the loss reserve at the beginning of the year.
3. Calendar/Accident year – Same earned premium as calendar year.
Losses are the amounts for accidents occurring during the year regardless of when reported.
b) Policy year – Advantage – Matches premiums & losses from policies.
Disadvantages – less mature then other methods.
Calendar Year – Advantage – Fully mature at end of the year.
Disadvantage – Premiums & Losses not matched well.
Cal/Acc Year – Advantage – Better job of matching premium to losses than Calendar Year.
Disadvantage – less mature then Calendar Year losses.

56. a. Under the current law, a worker with 60% of the SAW, $540, gets the minimum benefit, and a worker with 1.5 times the SAWW, $1350, gets the maximum benefit.
Thus 25% of workers get the minimum, and 1 - 90% =10% get the maximum.
The workers who get in between the minimum and the maximum have a portion of wages of:
82% - 13% = 69%.
Average current benefit is: (25%)(360) + (69%)(2/3)(900) + (10%)(900) = $594.
Average benefit as percentage of average weekly wage is: 594/900 = **66%**.
b. Under the proposed law, a worker with half the SAWW, $450, gets the minimum benefit, and a worker with 1.25 times the SAWW, $1125, gets the maximum benefit.
15% of workers would get the minimum, and 1 - 80% = 20% would get the maximum.
The workers who would get in between the minimum and the maximum have a portion of wages of:
67% - 7% = 60%.
Average proposed benefit is: (15%)(360) + (60%)(80%)(900) + (20%)(900) = $666.
The direct effect is: 666/594 - 1 = **12.1% increase**.

57. "First, higher unemployment may increase utilization of workers' compensation income benefits as workers without jobs seek to retain income from whatever sources are available. Some of those unemployed will make claims that they would not have otherwise made, and extend the durations of the claims as long as possible or until job opportunities surface. Some who are receiving benefits will find that they no longer have jobs to which they can return. They seek to extend the duration of benefits. Some with residual disabilities find that they are especially at a competitive disadvantage in the labor market when unemployment rises. In each of these instances, workers may use more medical care in their efforts to establish entitlement or retain benefits.
Second, when unemployment is higher, some employed workers with relatively minor injuries will be more reluctant to file workers' compensation claims, fearing that they may be more vulnerable to layoff if not currently working. When some minor claims are not brought, it makes the average costs of a claim - medical as well as indemnity - appear to be increasing, as the fraction of more serious cases rises.
And third, when unemployment rises, the experience and injury mix of employed workers changes. Less experienced workers are laid-off, and more experienced workers retained. Less experienced workers tend to be younger, and have more frequent, but less serious injuries. As a consequence, the average severity of injury and average medical costs -would increase."
Comment: Based on Feldblum, "Workers' Compensation Ratemaking," formerly on the syllabus.

58. E. AY 2000 is currently at 36 months (the time from 1/1/00 to 12/31/02).
AY 2000 has a current case incurred of: \$6000 + \$2000 = \$8000.
AY 2000 has a projected ultimate of \$9,240.
$\Rightarrow$ the loss development factor from 36 months to ultimate is: 9240/8000 = 1.155.
AY 2001 is currently at 24 months, with a case incurred of: \$3500 + \$4000 = \$7500.
Projected ultimate accident year 2001 losses are: (1.20)(1.155)(\$7500) = **\$10,395**.
Comment: "24-36 case-incurred link ratio" is the factor to develop reported losses at 24 months to 36 months, in other words from the level 2 years from the beginning of the accident year to the level 3 years from the beginning of the accident year.
Projected ultimate accident year 2002 losses are: (1.71)(1.20)(1.155)(\$5000) = \$11,850.

59. D. 1. True. 2. False. 3. True.

60. a. For example, \$1816/\$1412 = 1.2861. The age-to-ultimate development factor for accident year 2003 as of December 31, 2003 is: (1.2551)(1.0769)(1.0000) = **1.3517**.

Accident Year	12	24	36	48	Ultimate
2000	\$1412	\$1816	\$1993	\$1993	\$1993
2001	\$1624	\$2023	\$2137		\$2137
2002	\$1841	\$2271			\$2446
2003	\$2421				**\$3272**
2000	1.2861	1.0975	1.0000		
2001	1.2457	1.0564			
2002	1.2336				
Average	1.2551	1.0769	1.0000		
Factor to Ultimate	**1.3517**	1.0769	1.0000		

I have chosen to average all available link ratios. “The loss development method is based upon the assumption that claims move from unreported to reported-and-unpaid to paid in a pattern that is sufficiently consistent that past experience can be used to predict future development.”

b. The ultimate loss amount for accident year 2003 is:

(age to ultimate factor)(most recent reported amount) = (1.3517)(\$2421) = **\$3272**.

c. New rates will be in effect from 7/1/04 to 6/30/05. ⇒ The average date of writing is 1/1/05.

Annual policies, so the average date of loss = average date of writing + 6 months = 7/1/05.

The trend period for AY 2003 is 2 years, from 7/1/03 to 7/1/05.

The trended ultimate loss amount for accident year 2003 is: $(1.08^2)(.98^2)$(\$3272) = **\$3665**.

d. 1. It takes time for claims to be reported. As new claims are reported over time, the incurred losses increase. 2. It takes time for claims to be settled. The total amount eventually paid on a claim is rarely equal to the amount reserved (or the sum of payments made to date plus reserves.) As payments are made and reserves taken down over time, the incurred losses change.

3. Some claims that are believed to be closed are reopened. As claims are reopened and then settled over time, the incurred losses change.

e. Trending projects the losses from the average experience period to the midpoint of the exposure period. For example, the losses for Accident Year 2003 are projected to the policies written from 7/1/04 to 6/30/05. Accident Year 2003 losses are at a maturity of 12 months; multiplying them by the trend factor produces an estimate of what those losses would have been at 12 months if they had instead been from policies written from 7/1/04 to 6/30/05. One would in addition need to multiply by a loss development factor in order to estimate the level of losses expected to be reported at ultimate.

Even if all losses were paid on the same day they occurred, while there would be no loss development, there still would be a need to apply a (severity) trend factor to adjust for inflation.

If instead we expect no inflation, then there would be no need to apply any (severity) trend factor, but we would still apply a loss development factor to adjust early reports to ultimate.

Both trend and loss development factors are required and there is no overlap between them.

Comment: The data could be losses, or losses plus alae, either paid or paid plus case reserves.

61. Multiply each loss by 1.15.

Loss	Contribution to Layer 50 xs 50		Inflated Loss	Contribution to Layer 50 xs 50
37,000	0		42,550	0
47,000	0		54,050	4,050
64,000	14,000		73,600	23,600
93,000	43,000		106,950	50,000
Sum	57,000		Sum	77,650

The annual claims inflation rate in the layer $50,000 excess of $50,000 is:
77,650/57,000 - 1 = **36.2%**.

<u>Comment</u>: It is unclear from the question whether one should trend all of the losses to some common date, and if so what that date would be.
If one were to inflate all the losses to the middle of 2004:

Loss	Contribution to Layer 50 xs 50		Trend Period (months)	Inflated Loss	Contribution to Layer 50 xs 50
37,000	0		17	45,101	0
47,000	0		11.5	53,736	3,736
64,000	14,000		9	71,073	21,073
93,000	43,000		7	100,900	50,000
Sum	57,000			Sum	74,809

74,809/57,000 - 1 = 31.2%.

62. D. For State A, the average date of writing is 7/1/06. For annual polices, the average date of loss is 6 months later or 1/1/07.
For State B, the average (and only) date of writing is 1/1/06. For annual polices, the average date of loss is 6 months later or 7/1/06.
<u>Comment</u>: In State B some <u>new</u> business may be written throughout the year, in which case the average date of writing would be somewhat later than 1/1. However, one should not worry about such potential complications on a simple multiple choice question such as this.
One would trend losses to a date beyond January 1, 2006, and thus you should be able to quickly eliminate choice C.

63. Assume that each payment to an injured worker for lost wages will be 15% more than it was. Then the direct effect is that indemnity losses will increase by 15%.
There are likely to be indirect effects that increase the indemnity losses paid by insurers by more than 15%. Since benefits are higher, more Workers Compensation claims will be made, and the average time before injured workers return to work will also increase.

64. Taking averages of all of the available development factors:

PY	27	39	51	63	75	
1999				$7,320	$7,906	
2000			$6,921	$7,613	$8,261	
2001		$7,424	$8,908	$10,156	$10,917	
2002	$4,733	$6,266	$7,225	$8,091		
2003	$4,969	$6,361	$7,505			
2004	$3,926	$5,261				
2005	$5,044					
PY	27:39	39:51	51:63	63:75	75:Ultimate	
1999				1.080		
2000			1.100	1.085		
2001		1.200	1.140	1.075		
2002	1.324	1.153	1.120			
2003	1.280	1.180				
2004	1.340					
Average	1.315	1.178	1.120	1.080	1.050	

Expected ultimate losses for policy year 2003: (1.120)(1.080)(1.050)(7505) = **$9,532**.

65. PY1999: (1.05)(7906) = $8301. PY2000: (1.05)(8261) = $8674.
PY2001: (1.05)(10,917) = $11,463. PY2002: (1.08)(1.05)(8091) = $9175.
PY2004: (1.178)(1.12)(1.08)(1.05)(5261) = $7871.
PY2005: (1.315)(1.178)(1.12)(1.08)(1.05)(5044) = $9924.

66. a. The layer $50,000 excess of $100,000 is from $100,000 to $150,000.

Original Loss	Contribution to Layer	Inflated Loss	Contribution to Layer
75,000	0	82,500	0
100,000	0	110,000	10,000
125,000	25,000	137,500	37,500
150,000	50,000	165,000	50,000
Total	75,000		97,500

The losses in this layer increase from: 75,000 to 97,500. 97.5/75 - 1 = **30%**.

b. 1. For losses already penetrating the higher layer, such as $120,000, inflation impacts only the portion of the loss in the higher layer. The basic limits portion does not change.
This tends to make the higher layer inflation rate greater than the basic limit inflation rate.
2. For losses near the bottom of the higher layer, such as $100,000, inflation causes the losses to pierce the higher layer, resulting in increased frequency of losses contributing to this layer.
This tends to make the excess layer inflation rate greater than the basic limit inflation rate.
3. For losses that already contribute the width of the layer, such as $150,000, their contribution to the higher layer is unchanged by inflation.
This tends to make the higher layer inflation rate smaller than the basic limit inflation rate.

Comment: Here is a calculation of the inflation rate for claims limited to $100,000.

Original Loss	Capped Loss	Inflated Loss	Capped Inflated Loss
75,000	75,000	82,500	82,500
100,000	100,000	110,000	100,000
125,000	100,000	137,500	100,000
150,000	100,000	165,000	100,000
Total	375,000		382,500

382,500/375,000 - 1 = 1.9%.

Basic limit losses increase slower than the overall rate of inflation.

It is the case that losses excess of a limit, in other words a layer from L to ∞, always increase faster than the overall rate of inflation. However, the rate of inflation for a layer with an upper limit, such as $50,000 excess of $100,000, may be higher, lower, or the same as the overall rate inflation. The answer depends on the particular layer and the size of loss distribution. It is not clear whether the writer of this exam question was aware of this. Item 3 in my solution to part b may not have been an intended possible answer. In any case, only give two reasons in part b.

67. a. Policy Year: Premiums and losses on policies whose effective dates are in the given year.
Calendar Year written premium is that premium written during a given year.
Calendar Year earned premium is written premium plus the change in unearned premium reserves during the given year.
Paid Calendar Year losses are paid in that year.
Incurred Calendar Year losses are those paid in that year plus the change in loss reserves.
Accident Year or Calendar/Accident Year premiums are the same as Calendar Year premiums.
Accident Year losses are those on accidents that occurred during a given year.
b. Calendar Year data is most responsive, since it is available quickly.
Policy Year data is least responsive, since it takes a while for it to become available.
For example, for Policy Year 2006, the last policy will expire 12/31/07.
<u>Comment</u>: Basic ideas you have to know!

68. Loss development factors are based on the losses prior to accident year 2006. Since the initial case reserves were much higher for older accident years than for 2006, the incurred development factors will be lower than the expected amount of development on AY 2006.
Estimates of ultimate losses for AY 2006 will be biased downwards, resulting in a rate indication that will be too low. The resulting rates will be inadequate.
<u>Comment</u>: If the actuary was aware of this change, he could try to adjust the AY 2066 data as of first report to what it would have been under the old reserving scheme. Perhaps he would add $5000 times the number of open cases, or a little less. Then he could apply the loss development factors to these adjusted AY 2006 losses.
If instead the amount had been changed from $10,000 to $15,000, then the rate indication would have been too high. If instead the amount had been changed from for example $10,000 to $10,500, in order to keep up with inflation, then perhaps the rate indication would be okay.

69. a. Direct effects are a direct and obvious consequence of the benefit change.
For example, if the maximum benefit is increased, losses will automatically go up because those already at the maximum will get an increase in benefits.
b. Indirect effects arise from changes in claimant behavior that are a consequence of a benefit change under the Workers Compensation Law.
c. There will be both direct and indirect effects from the implementation of cost of living adjustments.
Direct: Indemnity payments will increase as they are adjusted upwards with inflation.
(Usually this would apply to injured workers who have received benefits for more than one year.)
Indirect: Some injured workers may stay out of work longer because their benefits are keeping up with inflation; they have less financial incentive to return to work than previously.
d. There could be an indirect effect but not a direct effect from changes in administrative procedures.
There could be an indirect effect if for example administrative procedures were changed to make it less complicated and burdensome to file claims. This would lead to more claims being filed.
Comment: Changes in interpretation of the law by the state department that is in charge of administering the Workers Compensation Law, (called in some states the Industrial Accident Board,) can act like a direct benefit effect. For example, they may change the guidelines for how to determine whether a worker is permanently and totally disabled. However, the term direct effect is usually reserved for changes in the law itself.

70. Compare the contributions to the layer from 100,000 to 200,000 before and after inflation:

Claim	Loss	Contribution to Layer	Inflated Loss	Contribution to Layer
A	35,000	0	37,800	0
B	125,000	25,000	135,000	35,000
C	180,000	80,000	194,400	94,400
D	206,000	100,000	222,480	100,000
E	97,000	0	104,760	4,760
Total		205,000		234,160

234,160/205,000 = 1.142. **14.2%** effective trend on this layer.
Comment: A small loss, such as 35,000, contributes nothing in both cases.
A larger loss, such as 97,000, increases its contribution from 0 to 4,760, increasing the effective trend in the layer.
A large loss, such as 206,000, contributed 100,000 in both cases, decreasing the effective trend in the layer.
Based on the combination of such impacts, the effective trend for a layer can be either more than or less than the ground-up unlimited trend.

71. a. Policy Year losses are allocated to the year in which the policy was written.
Calendar Year losses are allocated to the year in which payments were made and reserves were changed. Accident Year losses are allocated to the year in which the accident occurred.
b. Calendar year is the most recent and responsive because there is no delay due to developing losses.
c. Policy year matches premiums and losses best because the losses are generated by the same policies for which premium was collected.

72. a. As of the end of 2006, the first loss has a reserve of 5000, and nothing has been paid.
CY06 incurred losses: **$5000**.
As of the start of 2007 the first loss has a reserve of 5000.
During 2007, 500 + 3500 = 4000 has been paid on the first loss.
At the end of 2007 the first loss has a reserve of 2000.
On the first loss, CY07 incurred is paid plus change in reserve: 4000 + (2000 - 5000) = 1000.
During 2007, 5000 + 9000 = 14,000 has been paid on the second loss.
At the end of 2007 the second loss has a reserve of 1000.
On the second loss, CY07 incurred is: 14,000 + 1000 = 15,000.
CY07 incurred losses: 1000 + 15,000 = **$16,000**.
b. The first loss is part of AY06.
As of 12/13/08 there is no reserve and 500 + 3500 + 3000 = 7000 has been paid.
AY06 incurred loss as of 12/31/08: **$7000**.
The second loss is part of AY07.
As of 12/13/08 there is no reserve and 5000 + 9000 + 1000 = 15000 has been paid.
AY07 incurred loss as of 12/31/08: **$15,000**.
c. Both losses are part of PY06, since they are from policies written in 2006.
PY06 incurred loss as of 12/31/08: $7000 + 15,000 = **$22,000**.
Neither loss is part of PY07.
PY07 incurred loss as of 12/31/08: **$0**.
d. Policy year losses are from the same policies as the corresponding policy year premiums.
This better match between premiums and losses is an advantage compared to using calendar/accident year.
Policy year losses take longer to be reported than accident year losses; this is a disadvantage compared to using calendar/accident year. In other words, Policy Year losses are less mature and less responsive.
For example, as of 12/31/06 most of polices written in PY06 have yet to expire; many of the losses that will enter into PY06 have yet to occur. In contrast, as of 12/31/06 all of the losses that will enter into AY06 have occurred.

73. a. We increase each loss by 10%. We limit both sets of losses to $50,000.

Total Limits Earlier Year	Basic Limits Earlier Year	Total Limits Later Year	Basic Limits Later Year
50,000	50,000	55,000	50,000
70,000	50,000	77,000	50,000
90,000	50,000	99,000	50,000
110,000	50,000	121,000	50,000
20,000	20,000	22,000	22,000
340,000	220,000	374,000	222,000

Basic Limit Trend: 222/220 - 1 = **0.91%**.

b. The excess losses in the earlier year are: 340,000 - 220,000 = 120,000.

The excess losses in the later year are: 374,000 - 222,000 = 152,000.

Excess Limit Trend: 152/120 - 1 = **26.67%**.

c. If no losses are excess of the basic limit, both before and after inflation, then the excess losses will be zero in both years. In that case, the excess limit trend will be zero, less than the basic limit trend.

For example, if we had losses of 20,000 and 30,000, then the basic limit trend would be 10% while the excess trend would be 0.

Alternately, when loss trends are negative, the excess limit trend will be less than the basic limit trend.

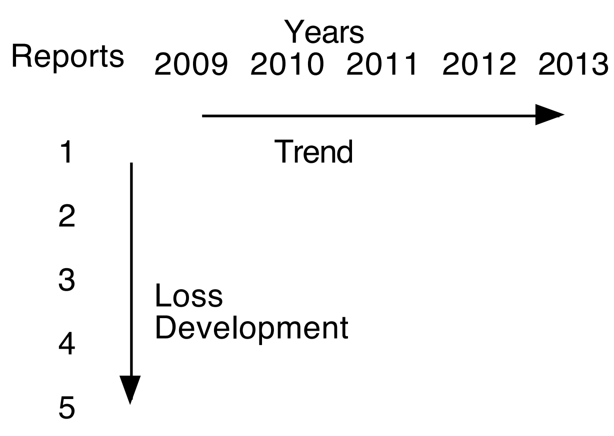
74. a. Under the prior law, a weekly wage of $100 results in the maximum benefit, while a wage of $75 results in the minimum benefit.
Thus 35% of workers receive the minimum benefit, while 1 - 60% = 40% of the workers receive the maximum benefit.
The remaining workers receive 2/3 of their weekly wage; they represent of the total wages:
42% - 20% = 22%.
Average weekly benefit under the prior law is:
(35%)($50) + (40%)($67) + (2/3)($100)(22%) = $58.97.
Under the new law, a weekly wage of $150 results in the maximum benefit, while a wage of $75 results in the minimum benefit.
Thus 35% of workers receive the minimum benefit, while 1 - 91% = 9% of the workers receive the maximum benefit.
The remaining workers receive 2/3 of their weekly wage; they represent of the total wages:
81% - 20% = 61%.
Average weekly benefit under the prior law is:
(35%)($50) + (9%)($100) + (2/3)($100)(61%) = $67.17
Direct benefit effect: $67.17/$58.97 - 1 = **13.9%**.
b. Indirect effects arise from changes in claimant behavior that are a consequence of a benefit change under the Workers Compensation Law.
c. Since high wage injured workers will receive more per week under the new law than under the old law, they have less incentive to return to work. Therefore, we would expect to see an increase in the average duration before workers return to work, resulting in higher average claim costs.
Also, since high wage injured workers will receive more per week under the new law than under the old law, they have more incentive to file a claim and go on Workers Compensation. Therefore, we might see an increase in the claim frequency.
<u>Comment</u>: $100 is much smaller than a realistic value of a statewide average weekly in 2008.

75. a) Calendar Year 2006 Paid Losses: 300,000 + 1,500,000 = **$1,800,000**.
Calendar Year 2006 Incurred Losses = Paid + Change in Reserves:
1,800,000 + (300,000 - 500,000) + (1,000,000 - 0) = **$2,600,000**.
b) Accident year incurred losses for 2006 evaluated as of December 31, 2007:
Cumulative Paid + Reserve = (1,500,000 + 700,000) + 200,000 = **$2,400,000**.
c) Policy year incurred losses for 2005 evaluated as of December 31, 2008:
Cumulative Paid + Reserve =
= 1,000,000 + 300,000 + 250,000 + 50,000 +1,500,000 + 700,000 + 100,000 + 0 + 50,000
= **$3,950,000**.
d) Calendar year incurred losses are more responsive than accident year, since calendar year losses are known and final at the end of the year.
Accident year incurred provides a better match between premiums and losses than calendar year, although not as good of a match as policy year.

76. Assume that losses are increasing due to inflation.
A loss that is greater than or equal to the basic limit, will not have its contribution to basic limits losses increase due to inflation.
Therefore, the basic limit trend is lower than the trend in total losses.
Applying a basic limits loss trend to total limits losses will underestimate the future total limit losses; the resulting estimate would be biased downwards.
Comment: Limited losses increase more slowly than the overall rate of inflation.
Excess losses increase more quickly than the overall rate of inflation.

77. 1) Claims have been shifting from health insurance to worker's compensation because unlike most health Insurance there are no deductibles or coinsurance in worker's compensation, and doctors are paid more for their services.
2) There has been an increase in attorney involvement, which increases the average duration of disability of injured workers, increases the average cost of case settlement, and may contribute to a higher frequency of claims being made.
Comment: Based on "Workers Compensation Ratemaking," by Sholom Feldblum, formerly on the syllabus.

78. The "overlap fallacy" incorrectly asserts that loss development and loss trend capture the same change in loss patterns, and therefore, using both would be "double counting".
In fact, loss development and trend are both needed; there is no overlap.
Loss trend projects losses from the midpoint of experience period to the midpoint of exposure period, while loss development adjusts immature losses to an ultimate level.



While these solutions are believed to be correct, anyone can make a mistake. If you believe you've found something that may be wrong, send any corrections or comments to:
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