1B, p. 72: (60%)(0.39) + (40%)(**0.75**) = 0.534.

1D, page 131, solution to the first Exercise:

$$\int_2^{2.5} \lambda(t)\, dt = \int_2^{2.5} 3t^2\, dt = t^3 \Big]_{t=2}^{t=2.5} = 7.625.$$

1E, page 183, Q. 14.8: In the first row and last column of the matrix it should be **0.05**.

1E, page 196, Q. 14.60: In the first row and last column of the matrix it should be **0.10**.

1L, p.412, sol. 15.17: **C.** (.7, .2, .1)Q = (.54, **.35**, **.11**) = distribution two years from now.
(54%)(.82%) + (35%)(9.07%) + (11%)(30.12%) = **6.93%**.

1L, p.413: **15.18. A., 15.19. E., & 15.20. B.** (1, 0, 0)Q = (.7, .2, .1) = distribution next year.
(1, 0, 0)Q^2 = (.7, .2, .1)Q = (.54, .35, .11) = distribution two years from now.
(1, 0, 0)Q^3 = (.54, .35, .11)Q = (.4525, .443, .1045) = distribution three years from now.
On average of the following three years: .7 + .54 + .4525 = **1.6925** are Good,
.2 + .35 + .443 = **0.993** are Typical, and .1 + .11 + .1045 = **0.3145** are Poor.

Stochastic Models Practice Exam #1, Q.4: B. At least 2%, but less than **3**%

Stochastic Models Practice Exam #1, Q.9:
If it has rained for the last two days, the probability that it will rain **tomorrow** is 80%.
If it rained today but not yesterday, the probability that it will rain **tomorrow** is 60%.
If it rained yesterday but not today, the probability that it will rain **tomorrow** is 50%.
If it has not rained in the last two days, the probability that it will rain **tomorrow** is 70%.

Stochastic Models Practice Exam #1, Solution 4: Letter choice is **B**.

2S, p.723, solution 34.14: $252 \int_0^1 y^4 (1 - y)^6\, dy$

3A, p. 30: $\frac{C(K_1) - C(K_2)}{K_2 - K_1} \geq \frac{C(K_2) - C(K_3)}{K_3 - K_2}$.

3A, p. 46, Q. 3.13: In addition to convexity being violated, the difference in premium between the 110 and 115 strike calls should be less than or equal to $5e^{-rT}$, which is less than 5.

3E, p. 180 near the top: ⇒ $\mathbf{p = \frac{exp[(\alpha - d)h] - d}{u - d}}$.

If $\alpha > r$, then $p = \frac{\exp[(\alpha - \delta)h] - d}{u - d} > \frac{\exp[(r - \delta)h] - d}{u - d} = p^*$.

3E, p. 194, 18.6: Chance of a 10 should be **30%**.
Solution then becomes: E[f(x)] = .3/1 + .4/5 + .3/10 = 0.41.
E[X] = (.3)(1) +(.4)(5) + (.3)(10) = 5.3. f(E[X]) = 1/5.3 = 0.1887 < 0.41.

3E, p. 198, half way down the page: If each X_i has mean μ and variance σ^2, then $\bar{X}_n$ has mean $\boldsymbol{\mu}$

3F, 2/3rds down page 217: $\text{Prob}[S_t > K] = 1 - \text{Prob}[S_t < K] = \Phi[\{\ln(S_0/K) + (\alpha - \delta - \sigma^2/2)t\}/(\sigma\sqrt{t})]$.

3F, p. 234, Q. 22.44:
Ignore the last line "You are given the following information for a stock with current price 0.25:"

3G, p. 266: delta in the numerator and r in the denominator, $d_1 = \frac{\ln[\frac{S_0\, e^{-\delta T}}{K\, e^{-rT}}] + \sigma^2 T / 2}{\sigma\sqrt{T}}$

3H, p. 319, exercise near the bottom: \$15 + (**2**)(.07) = \$15.14.

3H, p. 339, exercise at the bottom, add a comment:
<u>Comment</u>: In each case we apply a one -period model.
When computing Δ_0, we assume that the tree ends after only one period.

3K, page 424, add the following explanation:
An ordinary call only pays off when the stock price at expiration is greater than the strike price of 120. For a barrier H greater than 120, the Up and In Call will not be worth anything unless the stock price also has reached the barrier sometime during the next year. For example, for H = 125, for the Up and In Call to be worth anything, the stock price must have reached 125 during the next year. It is possible for the final stock price to be for example 123, but for the stock price to not have reached 125 during the year, however this is not very likely. In any case, much of the value of the ordinary call comes from large payoffs that occur when the final stock price exceeds 125. Thus for H = 125, the Up and In Call is worth 6.52, almost as much as the ordinary call premium of 6.53.

3R, p. 647, third line of the solution to the exercise: $\frac{\partial P}{\partial r}\, \boldsymbol{\sigma(r)}\, dZ$

3T, page 716, solution to the second exercise is wrong:
Exercise: Determine the call and put premiums for 2-Year 0.94-strike options on a zero-coupon bond with one more year to maturity.
(The options expire 2 years from now, while the bond matures 3 years from now.)
[Solution: The price of bond when the option expire is either $e^{-0.08} = 0.923116346$,
$e^{-0.06} = 0.941764534$, or $e^{-0.04} = 0.960789439$, with probabilities of: 25%, 50%, and 25%.

$$C = \frac{(0.25)(0.960789439 - .94)}{\exp[0.06 + 0.05]} + \frac{(0.25)(0.941764534 - .94)}{\exp[0.06 + 0.05]} + \frac{(0.25)(0.941764534 - .94)}{\exp[0.06 + 0.07]} =$$

0.005438512.

$$P = \frac{(0.25)(0.94 - 0.923116346)}{\exp[0.06 + 0.07]} = 0.003706365.$$

We discount the option payoffs back, where the middle node has two possible discount paths.]
$C_{Eur}(K, T) = P_{Eur}(K, T) + B_0 - PV[Coupons] - K e^{-rT}$, where e^{-rT} is the price of a zero-coupon bond that pays 1 at time T. For a zero-coupon bond, $C = P + B_0 - K e^{-rT}$.
Here $B_0 = 0.835479043$ = the price of the three year bond, the underlying asset of the options, and $e^{-rT} = 0.886964783$ = the price of the two year bond, the discount factor to time T = 2.
$C = 0.005438512 = 0.003706365 + 0.835479043 - (0.94)(0.886964783) = P + B_0 - K e^{-rT}$.
Verifying that put-call parity holds for this example.

3U, p. 741: $\frac{C(K_1) - C(K_2)}{K_{\mathbf{2}} - K_{\mathbf{1}}} \geq \frac{C(K_2) - C(K_3)}{K_{\mathbf{3}} - K_{\mathbf{2}}}$.

3U, p. 748, near the top: $p = \frac{\exp[(\alpha - \delta)h] - d}{u - d}$

3V, sol. 4.14: Buy a call with strike 100: cost 27. Sell a put with strike 100: get 25.
Sell a call with strike 120: get 23. Buy a put with strike 120: cost 26.
Cost is: 27 + 26 - 25 - 23 = 5.

3W, p. 819, sol. 11.25: and buy: (10,000)(45.20) = 45**2,0**00 bonds.
From shorting stock we get: (5881)(100) = 588,100.
Buying calls we spend: (13)(10,000) = 130,000. Buying bonds we spend: 452,000.
Therefore, setting up this portfolio, we get: 588,100 - 130,000 - 452,000 = 6100.
Thus, we can lend an additional 6100, so we have a total of: 452,000 + 6100 = 458,100 bonds.

3W, p. 840, sol. 14.30, line 6: K - **100** > (1 - .4625)(K - 86.94)/$e^{.01}$. ⇒ K > 114.85.

3W, p. 861, sol. 20.17: letter solution is **A**.

3X, p.865, sol. 22.8: **90%** confidence interval is:

The SOA/CAS added four more MFE/3F Sample Exam questions:
61. Would go in my Section 59.
62. Would go in my Section 61.
63. Would go in my Section 60. Also see my Section 55.
64. Would go in my Section 60.

61. Assume the Black-Scholes framework.
You are given:
(i) S(t) is the price of a stock at time t.
(ii) The stock pays dividends continuously at a rate proportional to its price.
The dividend yield is 1%.
(iii) The stock-price process is given by

$$\frac{dS(t)}{S(t)} = 0.05dt + 0.25dZ(t)$$

where {Z(t)} is a standard Brownian motion under the true probability measure.
(iv) Under the risk-neutral probability measure, the mean of Z(0.5) is -0.03.
Calculate the continuously compounded risk-free interest rate.
(A) 0.030 (B) 0.035 (C) 0.040 (D) 0.045 (E) 0.050

61. D. In the actual environment: $\frac{dS(t)}{S(t)} = (\alpha - \delta)dt + \sigma dZ(t)$.

$\Rightarrow \sigma = 25\%$, and $\alpha - \delta = 5\%$. We are given $\delta = 1\%$. $\Rightarrow \alpha = 6\%$.

In the risk neutral environment: $\frac{dS(t)}{S(t)} = (r - \delta)dt + \sigma d\tilde{Z}(t)$.

$\Rightarrow dZ(t) = d\tilde{Z}(t) - \frac{\alpha - r}{\sigma} dt = d\tilde{Z}(t) + 4(r - 6\%)\ dt$.

$\Rightarrow Z(t) = \tilde{Z}(t) + 4(r - 6\%)\ t$.

In the risk neutral environment, $d\tilde{Z}(t)$ is a standard Brownian Motion, and thus: $E^*[\tilde{Z}(t)] = 0$.
Therefore, $E^*[Z(t)] = 4(r - 6\%)t$.
$\Rightarrow -0.03 = E^*[Z(0.5)] = 4(r - 6\%)(0.5)$. $\Rightarrow$ r = **4.5%**.

Comment: In general, $\tilde{Z}(t) = Z(t) + \frac{\alpha - r}{\sigma} t$, where $\frac{\alpha - r}{\sigma}$ = Sharpe ratio.

Therefore, $E^*[Z(t)] = -\frac{\alpha - r}{\sigma}t$, and $E[\tilde{Z}(t)] = \frac{\alpha - r}{\sigma}t$.

62. Assume the Black-Scholes framework.
Let S(t) be the time-t price of a stock that pays dividends continuously at a rate proportional to its price. You are given:

(i) $\frac{dS(t)}{S(t)} = \mu dt + 0.4 d\tilde{Z}(t)$,

where $\{\tilde{Z}(t)\}$ is a standard Brownian motion under the risk-neutral probability measure;

(ii) for $0 \le t \le T$, the time-t forward price for a forward contract that delivers the square of the stock price at time T is

$$F_{t,T}(S^2) = S^2(t) \exp[0.18(T - t)].$$

Calculate μ.

(A) 0.01 (B) 0.04 (C) 0.07 (D) 0.10 (E) 0.40

62. A. In the risk neutral environment: $\frac{dS(t)}{S(t)} = (r - \delta)dt + \sigma d\tilde{Z}(t)$.

Therefore, $\sigma = 0.4$ and $\mu = r - \delta$.

$F_{t,T}(S^a) = S^a(t) \exp[\{a(r - \delta) + a(a-1)\sigma^2/2\} (T - t)]$.

$\Rightarrow F_{t,T}(S^2) = S^2(t) \exp[\{2(r - \delta) + \sigma^2\} (T - t)]$.

$\Rightarrow 0.18 = 2(r - \delta) + \sigma^2$.

$\Rightarrow \mu = r - \delta = \frac{0.18 - 0.4^2}{2} = \mathbf{0.01}$.

63. Define
(i) $W(t) = t^2$.
(ii) $X(t) = [t]$, where $[t]$ is the greatest integer part of t;
for example, [3.14] = 3, [9.99] = 9, and [4] = 4.
(iii) $Y(t) = 2t + 0.9Z(t)$, where $\{Z(t): t \geq 0\}$ is a standard Brownian motion.
Let $V_T^2(U)$ denote the quadratic variation of a process U over the time interval [0, T].
Rank the quadratic variations of W, X and Y over the time interval [0, 2.4].
(A) $V_{2.4}^2(W) < V_{2.4}^2(Y) < V_{2.4}^2(X)$
(B) $V_{2.4}^2(W) < V_{2.4}^2(X) < V_{2.4}^2(Y)$
(C) $V_{2.4}^2(X) < V_{2.4}^2(W) < V_{2.4}^2(Y)$
(D) $V_{2.4}^2(X) < V_{2.4}^2(Y) < V_{2.4}^2(W)$
(E) None of the above.

63. A.

(i) $W(t) = t^2$. $dW = 2t dt$. $dW^2 = 4 t^2 dt^2 = 0$.

Quadratic variation over the time interval 0 to 2.4 is:

$$\int_0^{2.4} \{dW(t)\}^2 = \int_0^{2.4} 0 = 0. \quad V_{2.4}^2(W) = \mathbf{0}.$$

(ii) $X(t) = 0$ for $0 \le t < 1$. Then it jumps up to 1 at one.

$X(t) = 1$ for $1 \le t < 2$. Then it jumps up to 2 at two.

$X(t) = 2$ for $2 \le t < 3$.

Thus for small time intervals, all of the increments are zero, except for those time intervals that include an integer, in which case the increment is 1.

Therefore, $\sum_{j=1}^{n} \{X(j2.4/n) - X((j-1)2.4/n)\}^2 = 1^2 + 1^2 = 2$.

Taking the limit as n approaches infinity, the quadratic variation of X is 2. $V_{2.4}^2(X) = \mathbf{2}$.

iii) $Y(t) = 2t + 0.9Z(t)$. $dY = 2dt + 0.9dZ(t)$.

$dY^2 = 4dt^2 + 3.6 \, dt \, dZ(t) + 0.81dZ(t)^2 = 0.81dt$.

Quadratic variation over the time interval 0 to 2.4 is:

$$\int_0^{2.4} \{dY(t)\}^2 = \int_0^{2.4} 0.81dt = (2.4)(0.81) = 1.944. \quad V_{2.4}^2(Y) = \mathbf{1.944}.$$

$$0 = V_{2.4}^2(W) < 1.944 = V_{2.4}^2(Y) < 2 = V_{2.4}^2(X).$$

64. Let S(t) denote the time-t price of a stock.

Let $Y(t) = [S(t)]^2$.

You are given

$$\frac{dY(t)}{Y(t)} = 1.2dt - 0.5dZ(t),\ Y(0) = 64,$$

where $\{Z(t): t \geq 0\}$ is a standard Brownian motion.

Let (L, U) be the 90% lognormal confidence interval for S(2).

Find U.

(A) 27.97 (B) 33.38 (C) 41.93 (D) 46.87 (E) 53.35

64. C. $dS = (\alpha - \delta)S\ dt + \sigma\ S\ dZ$. $dS^2 = \sigma^2\ S^2\ dt$.

$Y = S^2$. $\partial Y/\partial S = 2S^2$. $\partial^2 Y/\partial S^2 = 2S$. $\partial Y/\partial t = 0$.

Using Ito's Lemma:

$dY = \partial Y/\partial S\ dS + \partial^2 Y/\partial S^2\ dY^2/2 + \partial Y/\partial t\ dt = \{2(\alpha - \delta) + \sigma^2\}\ Y\ dt + 2\sigma\ Y\ dZ$

We are given $\frac{dY(t)}{Y(t)} = 1.2dt - 0.5dZ(t)$.

Therefore, $\sigma = -0.25$, and $2(\alpha - \delta) + \sigma^2 = 1.2$. $\Rightarrow \alpha - \delta = 0.56875$.

Y follows a Geometric Brownian Motion and therefore, so does S.

$Y(0) = 64$. $\Rightarrow S(0) = \sqrt{64} = 8$.

Therefore, S(2) is Lognormal with parameters:

$\ln(8) + \{0.56875 - (-0.25)^2/2\}(2) = 3.1544$, and $-0.25\sqrt{2} = -0.3536$.

Thus a 90% confidence interval for ln[S(2)] has endpoints:

$3.1544 \pm (1.645)(-0.3536) = 2.5727$ and 3.7361.

$e^{2.5727} = 13.10$. $e^{3.7361} =$ **41.93**.

Alternately, the stochastic differential equation for Y is that of a Geometric Brownian Motion.

$Y(0) = 64$.

Therefore, $Y(t) = 64 \exp[(1.2 - (-0.5)^2/2)t - 0.5Z(t)] = 64 \exp[1.075\ t - 0.5\ Z(t)]$.

$\Rightarrow S(t) = \sqrt{Y(t)} = 8 \exp[0.5375\ t - 0.25\ Z(t)]$.

$\Rightarrow S(2) = 8 \exp[1.075 - 0.25\ Z(2)]$.

Z(2) is Normal with mean zero and standard deviation $\sqrt{2}$.

Therefore, a 90% confidence interval for S(2) has endpoints:

$8 \exp[1.075 - (0.25)(-1.645)\sqrt{2}] =$ **41.93**, and $8 \exp[1.075 - (0.25)(1.645)\sqrt{2}] = 13.10$.

<u>Comment</u>: I do not know why sigma is negative!

Nor do I know why $\alpha - \delta$ is about 57%.

If S follows a Geometric Brownian Motion, then S^a also follows a Geometric Brownian Motion.

This follows from applying Ito's Lemma, or that $S(t) = S(0) \exp[(\alpha - \delta - \sigma^2/2)t + \sigma\ Z(t)]$, and

therefore, $S(t)^a = S(0)^a \exp[a\ (\alpha - \delta - \sigma^2/2)\ t + a\ \sigma\ Z(t)]$.

If $C = S^a$, then $dC / C = \{a(\alpha - \delta) + a(a-1)\sigma^2/2\}\ dt + a\sigma\ dZ$.

For $Y = S^2$, $dY/Y = \{2(\alpha - \delta) + \sigma^2\}\ dt + 2\sigma\ dZ$.

Not yet in Study Guide

Assume the Black-Scholes framework.

Let S(t) be the time-t price of a stock that pays dividends continuously at a rate proportional to its price.

For $0 \le t \le T$, the time-t forward price for a forward contract that delivers the cube of the stock price at time T is: $F_{t,T}(S^3) = S^3(t)\ \exp[0.43(T - t)]$.

$r = 4.4\%$. $\delta = 1\%$.

Determine σ.

(A) 21% (B) 24% (C) 27% (D) 30% (E) 33%

E. $F_{0,T}[S_T^a] = S_0^a \exp[\{a (r - \delta) + a (a-1) \sigma^2 / 2\} T]$.

$F_{t,T}[S^3] = S^3(t) \exp[\{3 (r - \delta) + 3 \sigma^2\} (T-t)]$.

$\Rightarrow 3 (r - \delta) + 3 \sigma^2 = 0.43$. $\Rightarrow \sigma =$ **33%**.

Comment: Similar to MFE Sample Q. 62.

Not yet in Study Guide

Assume the Black-Scholes framework.

You are given:

(i) S(t) is the price of a stock at time t.

(ii) The stock pays dividends continuously at a rate proportional to its price.
The dividend yield is 2%.

(iii) The stock-price process is given by

$$\frac{dS(t)}{S(t)} = 0.1dt + 0.4dZ(t)$$

where {Z(t)} is a standard Brownian motion under the true probability measure.

(iv) The continuously compounded risk-free interest rate is 4%.

Under the risk-neutral probability measure, determine the mean of Z(3).

(A) -0.6 (B) -0.5 (C) -0.4 (D) -0.3 (E) -0.2

A. In the actual environment: $\frac{dS(t)}{S(t)} = (\alpha - \delta)dt + \sigma dZ(t)$.

$\Rightarrow \sigma = 40\%$, and $\alpha - \delta = 10\%$. We are given $\delta = 2\%$. $\Rightarrow \alpha = 12\%$.

In the risk neutral environment: $\frac{dS(t)}{S(t)} = (r - \delta)dt + \sigma d\tilde{Z}(t)$.

$\Rightarrow dZ(t) = d\tilde{Z}(t) - \frac{\alpha - r}{\sigma} dt$.

$\frac{\alpha - r}{\sigma} = \frac{12\% - 4\%}{40\%} = 0.2. \Rightarrow Z(t) = \tilde{Z}(t) - 0.2\ t$.

In the risk neutral environment, $d\tilde{Z}(t)$ is a standard Brownian Motion, and thus: $E^*[\tilde{Z}(t)] = 0$.

Therefore, $E^*[Z(t)] = -0.2\ t$.

$E^*[Z(3)] = (-0.2)(3) =$ **-0.6**.

Comment: Similar to MFE Sample Q. 61.

In general, $\tilde{Z}(t) = Z(t) + \frac{\alpha - r}{\sigma} t$, where $\frac{\alpha - r}{\sigma}$ = Sharpe ratio.

Therefore, $E^*[Z(t)] = -\frac{\alpha - r}{\sigma}t$, and $E[\tilde{Z}(t)] = \frac{\alpha - r}{\sigma}t$.