Microeconomics, elasticity, final exam practice problems

** Exercise 4.1: Elasticities

A perfectly competitive industry has identical firms and identical consumers. Each consumer earns $10,000 a year.

- The demand curve is Q = 100 – 5P.
- The supply curve is Q = 20 + 3P.

A. What is the equilibrium price?
B. What is the equilibrium quantity?
C. What is the price elasticity of demand at the equilibrium price?
D. What is the price elasticity of supply at the equilibrium price?
E. If consumers earn $11,000 a year, their demand curve changes to Q = 104 – 4P. What is the income elasticity of demand at an income of $10,000?

Part A: Solve for the equilibrium price as 100 – 5P = 20 + 3P ⇒ 8P = 80 ⇒ P = 10.

Part B: Solve for the equilibrium quantity as 100 – 5 × 10 = 50.

Part C: The price elasticity of demand is –5 × 10 / 50 = –1.

Part D: The price elasticity of supply is +3 × 10 / 50 = +0.600.

Part E: The percentage change in income is $1,000 / $10,000 = 10%.

- The new equilibrium price is 104 – 4P = 20 + 3P ⇒ 7P = 84 ⇒ P = 12.
- The new equilibrium quantity is 104 – 4 × 12 = 56.

The income elasticity of demand is ((56 – 50) / 50) / 10% = +1.20.

Jacob: Is the income elasticity of demand exactly 1.2 at an income of $10,000?

Rachel: The elasticity is for a point: it is ∂Q/Q ÷ ∂Y/Y at a specific income Y. This exercise gives the change in quantity for a large change in the consumer's income. The income elasticity of demand changes at each income level, so the solution is only an approximation. A more exact solution uses the average quantity and the average income in the range for the income elasticity of demand at the midpoint of the range, giving

((56 – 50) / (½ × (50 + 56)) / (($11,000 – $10,000) / ½ × ($10,000 + $11,000)) = +1.189.

For final exam problems, use the approximation in Part E of this exercise: (new Q – old Q) / old Q ÷ (new Y – old Y) / old Y. This will give one of the multiple choice solutions.

Jacob: Does this problem affect the price elasticity of demand and the price elasticity of supply?

Rachel: The exercise gives the supply and demand curves, so differentiation gives exact solutions. If we express the demand curve as a function of income, we could derive the exact income elasticity of demand.

Jacob: Is the new demand curve consistent with a higher income?

Rachel: The coefficient of P is less negative: –4 instead of –5, meaning that the consumer is less concerned about the price of the good. This makes sense: the consumer has more income and is more willing to pay a higher price. The equilibrium quantity is higher: 56 instead of 50, so this is a normal good.

One might say that the constant in a linear demand curve should not change. If the consumer buys 100 at a price of zero at income of $10,000, he should buy 100 at a price of zero at income of $11,000. But this is not a serious objection. The demand curve is not really linear at a price of zero; the linear demand curve is just an approximation of demand at existing market prices. This exercise uses an intercept of 104 instead of 100 for an income of $11,000 to simplify the mathematics.

** Exercise 4.2: Price Elasticity of Demand

The demand curve is linear: $Q = \alpha - \beta \times P$.

- At a price of $P = 0$, consumers buy 100 units of Q.
- At a price of $P = 100$, the price elasticity of demand is –1.

A. What is the value of α?
B. What is the price elasticity of demand at a price P as a function of β?
C. What is the value of β?
D. What is the price elasticity of demand at a price of 50?

Part A: $Q = \alpha - \beta \times 0 = 100 \Rightarrow \alpha = 100$

Part B: η = the price elasticity of demand = $\partial Q / \partial P \times P/Q = -\beta \times P / (100 - \beta \times P)$.

Part C: $-1 = -\beta \times 100 / (100 - \beta \times 100) \Rightarrow$

$-1 = -(100 - \beta \times 100) / \beta \times 100 \Rightarrow -1 / \beta + 1 \Rightarrow \beta = ½$

Part D: η = the price elasticity of demand = $\partial Q / \partial P \times P/Q = -½ \times 50 / (100 - ½ \times 50) = -0.333 = -⅓$

** Exercise 4.3: Price Elasticity of Demand

The demand curve is linear: $Q = \alpha - \beta \times P$.

- At a price of $P = 20$, the price elasticity of demand is $-\frac{2}{3}$.
- At a price of $P = 30$, the price elasticity of demand is -1.5.

A. What is the price elasticity of demand at a price P as a function of α and β?
B. What are the values of α and β?
C. What is the price elasticity of demand at a price of 40?

Part A: η = the price elasticity of demand = $\partial Q / \partial P \times P/Q = -\beta \times P / (\alpha - \beta \times P)$.

Part B: We have two equations in two unknowns:

$-1.5 = -\beta \times 30 / (\alpha - \beta \times 30) \Rightarrow 2.5 \times 30\ \beta = 1.5 \times \alpha$
$-\frac{2}{3} = -\beta \times 20 / (\alpha - \beta \times 20) \Rightarrow 1\frac{2}{3} \times 20\ \beta = \frac{2}{3} \times \alpha$

Solving gives $\alpha = 100$ and $\beta = 2$.

Part C: η = the price elasticity of demand = $\partial Q / \partial P \times P/Q = -2 \times 40 / (100 - 2 \times 40) = -4.000$

** Exercise 4.4: Income Elasticity of Demand

The Engel curve for wine is $Q = 4 \times \ln(Y - 6)$, where Y is the consumer's income in thousands of dollars (for $Y > 7$) and Q is quantity in flasks of wine.

A. What is the formula for this consumer's income elasticity of demand for wine?
B. What is the income elasticity of demand at an income of $31,000?

Part A: The income elasticity of demand is $\partial Q/\partial Y \times Y/Q =$

$$[4 / (Y - 6)] \times [Y / (4 \times \ln (Y - 6))] = Y / [(Y - 6) \times \ln (Y - 6)].$$

Part B: At $Y = 31$, this is $31 / [25 \ln (25)] = 0.385$

Jacob: Is this Engel curve realistic?

Rachel: This curve is chosen because it is easy to differentiate. It assumes that wine is a normal good for all people, but that people have a maximum amount of wine that they can consume. (Whether that is realistic, we don't know.) As increase increases without bound, the income elasticity of demand stays positive but approaches zero. As increase decreases, people buy less wine (since they have less income), but more of each dollar of additional income is spent on wine.

** Exercise 4.5: Income Elasticity of Demand

David and Jonathan have the same tastes. They differ only in their income and wealth. In 20X7

- Jonathan has royal income of 20,000 shekel a year and wealth of 100,000 shekel.
- David has a soldier's income of 2,000 shekel a year and wealth of 10,000 shekel.

Pita bread is the cheapest food sold, but it does not taste good. Other kinds of bread taste better, but they are more expensive.

In 20X8, both David and Jonathan earn twice as much as they did in 20X7. Which of the following are possible results?

A. David and Jonathan both buy less pita bread.
B. David and Jonathan both buy more pita bread.
C. David buys more pita bread and Jonathan buys less pita bread.
D. David buys less pita bread and Jonathan buys more pita bread.

Part A: We are not told how much pita bread costs. Perhaps all bread is subsidized by the king, so both David and Jonathan buy more expensive bread instead of pita. In this scenario, pita is an inferior good even for David (and surely for Jonathan).

Part B: We are not told how much pita bread costs. Perhaps there is a famine in the land, and all bread is imported from Moab. Both David and Jonathan can't afford more expensive bread, so they buy more pita. In this scenario, pita is a normal good even for Jonathan and surely for David.

Part C: Perhaps David can afford only pita bread but Jonathan can afford more expensive bread. When their incomes rise, David buys more pita bread and Jonathan buys less pita bread (and more of other types).

Part D: As income rises, some normal goods become inferior goods; the opposite does not happen. This last scenario is not reasonable.

** Exercise 4.6: Cross Elasticities

Are the following statements about elasticities and cross elasticities true or false?

A. The price elasticity of demand is likely to be highly elastic if there are a large number of close substitutes.
B. The cross elasticity of demand for *substitutes* is positive.
C. High cross elasticities are evidence of competition and low cross elasticities are evidence of monopoly.
D. The cross elasticity of demand for *complements* is negative.
E. As trade barriers fall and more firms compete, cross elasticities fall.

Answer 4.6: E

Part A: True. The price elasticity of demand is highly negative (= highly elastic) if there are many substitutes. An increase in price leads consumers to use substitutes, so the quantity demanded falls.

Part B: True: A rise in the price of one substitute causes consumers to demand more of the other substitute. That is, if the price of one good rises, more of the substitute good is demanded.

Part C: True. High cross elasticity means many close substitutes and little market power. If one firm raises the price for its product, consumers will buy the substitute goods from other suppliers.

Jacob: What if the monopoly produces all the substitutes?

Rachel: Substitutes refers to goods made by different firms. If one firm makes blue autos and red autos, the autos are two versions of the same good, not substitute goods.

Part D: True. Large cars and gasoline are complements. If the price of gas rises, consumer buy fewer large cars (and perhaps more hybrid cars) and are more likely to rise buses and trains.

Part E: False. As statement C says, *high* cross elasticities indicate more competition. As trade barriers fall and more firms make competing products, consumers can switch suppliers more easily.

** Exercise 4.7: Cross Elasticities

Microsoft and Sony make similar video game consoles named M and S.

- Firm N makes games (called N) for the Microsoft console.
- Firm T makes games (called T) for the Sony console.

The relations among these products are

- Consoles M and S are close substitutes.
- M and N are complements; S and T are complements.

As the price of Microsoft consoles (M) increases, what are the effects on the quantity sold of

A. Microsoft consoles
B. Sony consoles
C. Firm N's games
D. Firm T's games

Part A: The price elasticity of demand is negative. As the price of Microsoft consoles increases, the quantity sold decreases.

Part B: Microsoft consoles and Sony consoles are close substitutes. If Microsoft consoles become more expensive, consumer buy more Sony consoles instead.

Part C: Microsoft consoles and firm N's games are complements. If consumers buy fewer Microsoft consoles, they buy fewer games from firm N.

Part D: Sony consoles and firm T's games are complements. If consumers buy more Sony consoles, they buy more games from firm T.

** Exercise 4.8: Short and long-run elasticities

A. For firms in a competitive industry, which is more elastic: the short- or long-run supply curve?
B. For competitive industry as a whole, which is more elastic: the short- or long-run supply curve?

Part A: The firm's long-run supply curve is more elastic. In the short run, the firm can not change the amount of capital used. If demand increases, the firm must meet the higher demand with production from the same factory, running night shifts and paying workers overtime. It is expensive to pay overtime wages, and factories have more down-time when they are run at too high a percentage of capacity. In the long-run, the firm can build a second factory and hire additional workers.

Part B: The industry's long-run supply curve is more elastic. In the short run, the supply curve for the industry is the sum of the supply curves for the firms. In the long-run, new firms may enter the industry. For a constant cost industry, the supply curve may be perfectly elastic.

** Exercise 4.9: Linear Demand Curves and Elasticities

The demand curve is $Q = 400 – 10P$. The marginal cost curve is constant at $MC = k$, where $0 < k < 40$ (k is a constant between 0 and 40) and does not differ between competitive and monopolistic production.

Let η be the price elasticity of demand at the equilibrium price in a competitive market and η' be the price elasticity of demand at the equilibrium price in a monopolistic market.

A. What is the range for the price elasticity of demand at the competitive price?
B. What is the range for the price elasticity of demand at the monopoly price?
C. What is the relation of the price elasticity of demand at the competitive price and the monopoly price?

Part A: The price elasticity of demand is negative; it may range from 0 to $-\infty$.

Part B: The price elasticity of demand at the monopoly price is less than –1. If it were between –1 and 0, the monopoly would raise the price and reduce the quantity, increasing revenue and reducing cost, thereby raising profit.

Part C: If the demand curve is linear and the marginal cost curve is constant, the monopoly quantity is two thirds of the competitive quantity. Let P^* and Q^* be the competitive price and quantity, and P^{**} and Q^{**} be the monopoly price and quantity. Then $\partial Q/\partial P$ is the same at all prices, and $\eta' / \eta = (P^{**} / P^*) \times (Q^* / Q^{**}) =$

$1.50 \times (40 – 0.1 \times Q^*) / (40 – 0.1 \times ⅔ \times Q^*)$

** Exercise 4.10: Elasticities

Would each of the following likely be a positive or a negative number?

A. The price elasticity of demand for cheap generic cigarettes bought by low-income persons.
B. The price elasticity of demand for expensive brand-name cigarettes bought by high-income persons.
C. The income elasticity of demand of middle-income persons (who buy both generic cigarettes and brand-name cigarettes) for generic cigarettes.
D. The cross elasticity of demand between generic cigarettes and disposable lighters. (Assume that one needs a lighter for cigarettes; matches have not yet been invented.)
E. The cross elasticity of demand between generic cigarettes and brand-name cigarettes.

Parts A and B: The price elasticity of demand is always negative.

Part C: The income elasticity of demand is positive for normal good and negative for inferior goods. Wealthy persons buy fewer generic cigarettes as their incomes rise (inferior good), and poor persons buy more generic cigarettes as their incomes rises (normal good). For middle-income persons, the income elasticity of demand for generic cigarettes may be positive or negative.

Part D: If lighters cost more (and no matches are available), poor persons buy fewer generic cigarettes. (For higher-income persons who use expensive lighters, the cost of disposable lighters does not affect cigarette consumption.)

Part E: As brand-name cigarettes cost more, middle-income persons buy more generic cigarettes, and as generic cigarettes cost more, they buy more brand-name cigarettes. The cross elasticity is positive.

** Exercise 4.11: Constant Price Elasticity of Demand

The demand curve is $P \times Q = k$, where k is a constant.

A. What is $\partial Q/\partial P$?
B. What is the price elasticity of demand for this good at any price P?

Part A: Write $Q = k / P \Rightarrow \partial Q/\partial P = -k/P^2$.

Part B: The elasticity $\eta = -k/P^2 \times (P/Q) = -k/P^2 \times (P/(k/P)) = -1$.

This demand curve has constant elasticity of –1 for all values of P.

** Exercise 4.12: Income Elasticity of Demand

- The cost of a loaf of bread rises from $2.00 in 20X1 to $2.20 in 20X2.
- A consumer's income rises from $20,000 a year in 20X1 to $24,200 a year in 20X2.
- The consumer buys 200 loaves of bread in 20X1 and 231 loaves of bread in 20X2.
- Inflation is 10% per annum for all goods.

A. What is the inflation-adjusted percentage change in the consumer's income from 20X1 to 20X2?
B. What is the inflation-adjusted percentage change in the price of a loaf of bread from 20X1 to 20X2?
C. What is the percentage change in the quantity of bread bought by the consumer from 20X1 to 20X2?
D. What is the consumer's income elasticity of demand for bread, assuming their quality does not change?

Part A: The inflation-adjusted percentage change in the consumer's income from 20X1 to 20X2 is

(48,400 / 1.100) / 40,000 – 1 = 10.00%

Part B: The inflation-adjusted percentage change in the price of a loaf of bread from 20X1 to 20X2 is

(2.00 / 1.100) / 2.200 – 1 = 0.000

Jacob: Why must we know the inflation-adjusted change in the price of a loaf of bread? We don't use that figure to compute the income elasticity of demand.

Rachel: If the real price of a loaf of bread changes, the change in the number of loaves purchased reflects both the income elasticity of demand and the price elasticity of demand. Since the real price of a loaf of bread does not change in this exercise, the change in the quantity purchased reflects only the income elasticity of demand.

Part C: The percentage change in the quantity of bread bought from 20X1 to 20X2 is

231 / 200 – 1 = 15.50%

Part D: The income elasticity of demand for bread (for this consumer) is

+15.5% / +10.0% = 1.550